Sample Final Exam Name:____________________

Use colored paper provided for your solution showing all work. At the end of the exam it should be stapled to the question sheet which should have your name. Please write everything pertaining to the same problem in one place if possible.

No calculators or other aids of any kind. Show all work.

The total possible number of points is 115, 105 may be considered a perfect score.

(1) If $A(2,0,3)$, $B(-1,2,0)$, $C(0,3,1)$ determine the point D such that A, B, C, D are the vertices of a parallelogram with sides AB, BC, CD and DA. [7]

(2) If $A(1,2,0)$, $B(2,0,-3)$, $C(0,3,5)$ (a) determine the area of the triangle with vertices A, B, C, (b) determine an equation of the plane passing through A, B, C. [12]

(3) Determine an equation of the plane through $(1,2,4)$ orthogonal to the line with symmetric equations $(x-1)/3 = (y+1)/4 = -z$. [5]

(4) For the curve with parametric equations $x = t - \sin t$, $y = 1 - \cos t$ (a) determine the curvature at each point, (b) assuming t means time, determine the tangential component of the acceleration at the point corresponding to $t = \pi/4$. [12]

(5) An object is projected from the origin into the positive quadrant of the xy-plane at speed 64 ft/s and at an angle of $\pi/3$ with the x-axis. From the fact that its acceleration is the constant vector of magnitude 32 ft/s^2 in the negative y-direction find its position at any time $t \geq 0$. Also find the highest position and the time at which it occurs. [8]

(6) Determine an equation of the tangent plane to the surface $z = 10(x-y)/(x+y)$ at the point on it where $x = 7$, $y = 3$. [7]

(7) Assuming $z = z(x,y)$ is implicitly defined as function of x, y by $xy^3 + yz^3 + zx^3 = 65$ (a) determine $\partial z/\partial x$ for all possible (x,y) and (b) assuming $z(1,2) = 3$ determine $\partial z/\partial x(1,2)$. [8]

(8) If $f(x,y) = (4 - x^2 - y^2)y$ find all critical points and for each critical point determine whether it yields a local maximum, local minimum or saddle point. [11]

(9) Determine the point in the first octant and on the surface $(1/x) + (1/y) + (8/z) = 1$ whose distance from $O(0,0,0)$ is least. Hint: minimize (distance)^2 instead of distance. [9]

(10) Evaluate $\iiint_E xyz\sqrt{4 - x^2 - y^2 - z^2}\, dV$ if $E = \{(x,y,z) \mid x^2 + y^2 + z^2 \leq 4, x, y, z \geq 0\}$. [9]

(11) Evaluate $\iint_R e^{\frac{x-y}{x+y}}\, dA$ where R is the triangular region with vertices $(0,0)$, $(1,0)$, $(0,1)$ by making a suitable change of variables. [10]

(12) Use Green's theorem to find the amount of work done by the force $\hat{F}(x,y) = -x^2(y+1)\hat{\imath} + y^2(x-1)\hat{\jmath}$ in moving a particle from the origin along the x-axis to $(2,0)$, then along the line segment from $(2,0)$ to $(0,1)$ and then back to the origin along the y-axis. [8]

(13) If C denotes the positively oriented boundary of the surface S which is the part of $x^2 + y + z = 4$ contained in the first octant with upward orientation

and $\hat{F} =< y, 2z, 3x >$ use Stokes' theorem to evaluate $\int_C \hat{F} \cdot d\hat{r}$ in terms of a surface integral. [11]