Math 454 Summer 2005 Due Wednesday 7/13/05

Homework #2

Counting problems:

1 How many permutations of $\{1, 2, 3, \ldots, 12\}$ are there that don't begin with 2?

2 How many permutations of $\{1, 2, 3, \ldots, 12\}$ are there that don't begin with 2 *and* don't end with 9?

3 In how many ways can seven people be seated in a circle if two arrangements are considered the same whenever each person has the same neighbors (but not necessarily on the same side)?

4 How many 11-permutations of the multiset

$$\{3 \cdot a, 4 \cdot b, 5 \cdot c\}$$

are there?

5 List all 3-combinations and 4-combinations of the multiset $\{2 \cdot a, 1 \cdot b, 3 \cdot c\}$.

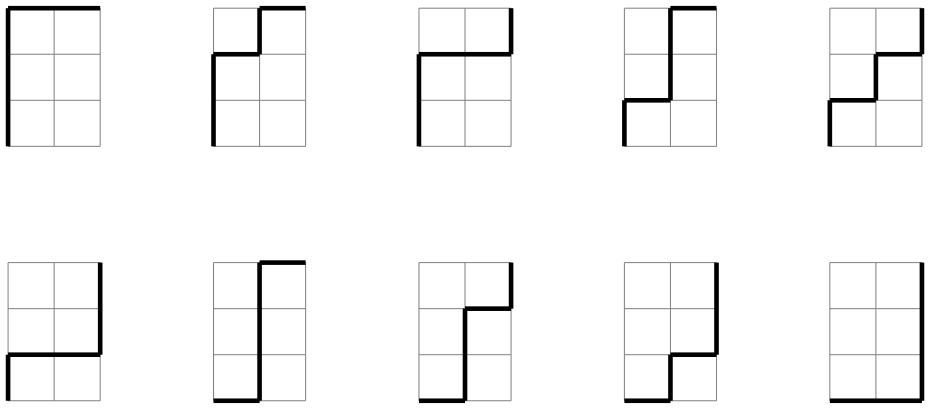
6 A bakery sells 6 different kinds of pastry. If the bakery has virtually unlimited supply of each kind, how many different options for a dozen of pastry are there? What if a box is to contain at least one of each kind of pastry?

7 How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

are there that satisfy $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -5$, and $x_4 \geq 8$?

8 How many paths are there from the point $(0, 0)$ to the point (m, n) if each step is either $(1, 0)$ or $(0, 1)$, or in other words if each step is either one unit east or one unit north? For example, the 10 paths of this type from $(0, 0)$ to $(2, 3)$ are shown below.



Hint: denote each $(1, 0)$ step by E and each $(0, 1)$ step by N, so each path corresponds to a sequence of E's and N's.

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$\boxed{\star 9}$ Prove that $\displaystyle\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$.

Hint #1: look over our combinatorial proof that $\displaystyle\sum_{k=0}^{n}\binom{n}{r} = 2^n$.

Hint #2: by the multiplication principle, one thing the right-hand-side of this identity counts are pairs (x, S) where S is a subset of $\{1, 2, 3, \ldots, n\}$ and x is a distinguished element of S.

Note: there is also a great probabilistic proof of this identity which begins by dividing both sides by 2^n. Then the right-hand-side is $n/2$, or in other words, the average size of a subset of $\{1, 2, 3, \ldots, n\}$ (why?). What's the left-hand-side?