

## Quiz #3 Solutions

- 1 A manager for a large corporation must prepare a list of projects that her group will complete over the next year. She has under consideration 6 such projects but will not be able to do all of them because of limits on personnel and budget. She has assigned a weight to each project that represents to her the value of completing the project. The personnel, capital requirements, and values for each project are given in the following table.

	Project					
	1	2	3	4	5	6
Person-weeks	250	195	200	70	30	40
Cost (in thousands of \$)	400	300	350	100	70	70
Value of completion	70	50	60	20	10	20

The manager has available 500 person-weeks and \$875,000 to allocate among the projects. Which projects should she choose to complete to maximize the value?

Formulate this problem as an integer programming problem, but do **not** attempt to solve it.

**Answer:** First we need to define our variables. I will choose

$$x_i = \begin{cases} 1 & \text{if the manager chooses to complete project } i, \\ 0 & \text{otherwise.} \end{cases}$$

We want to maximize total value, so this is

$$70x_1 + 50x_2 + 60x_3 + 20x_4 + 10x_5 + 20x_6.$$

Then we have two constraints, the manpower constraint

$$250x_1 + 195x_2 + 200x_3 + 70x_4 + 30x_5 + 40x_6 \leq 500,$$

and the cost constraint

$$400x_1 + 300x_2 + 350x_3 + 100x_4 + 70x_5 + 70x_6 \leq 875.$$

(Note that in the above we had 875, instead of 875,000, because the cost of completing the projects was given in thousands of dollars.) We also have to add constraints that say that each project can only be completed once:

$$x_i \leq 1, \text{ for } i = 1, 2, \dots, 6.$$

Our integer programming problem is then

$$\text{Maximize } z = 70x_1 + 50x_2 + 60x_3 + 20x_4 + 10x_5 + 20x_6$$

subject to

$$250x_1 + 195x_2 + 200x_3 + 70x_4 + 30x_5 + 40x_6 \leq 500$$

$$400x_1 + 300x_2 + 350x_3 + 100x_4 + 70x_5 + 70x_6 \leq 875$$

$$x_i \leq 1 \text{ for } i = 1, 2, \dots, 6$$

$$x_i \geq 0, \text{ and integral, for } i = 1, 2, \dots, 6$$