Math 354 Summer 2004

Quiz #1 Solutions

Please show your work and feel free to use the back of the sheet if you need more room.

1 Use Gauss-Jordan reduction to solve the following system of equations for x, y, and z:

$$\begin{array}{rcrcrcl} x & + & y & + & 2z & = & 8, \\ -x & + & y & + & 2z & = & 6, \\ 5x & & & + & z & = & 7. \end{array}$$

Answer: First we convert this into a linear algebra problem. Thus we want to solve $A\underline{x} = \underline{b}$ where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 8 \\ 6 \\ 7 \end{bmatrix}.$$

So we have to use Gauss-Jordan reduction on the augmented matrix

$$\left[\, A \,\middle|\, \underline{b} \,\right] = \left[\begin{array}{rrr|r} 1 & 1 & 2 & 8 \\ -1 & 1 & 2 & 6 \\ 5 & 0 & 1 & 7 \end{array}\right].$$

Your steps may vary, but here's how I did it:

$$\left[\begin{array}{rrr|r} 1 & 1 & 2 & 8 \\ -1 & 1 & 2 & 6 \\ 5 & 0 & 1 & 7 \end{array}\right] \to \left[\begin{array}{rrr|r} 1 & 1 & 2 & 8 \\ 0 & 2 & 4 & 14 \\ 5 & 0 & 1 & 7 \end{array}\right] \to \ldots \to \left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

(That was sortof a joke.) Now you can translate back into a system of equations and get

$$\begin{array}{rcl} x & = & 1, \\ y & = & 3, \\ z & = & 2. \end{array}$$

2 Compute the inverse of $A = \begin{bmatrix} 5 & 3 \\ 5 & 2 \end{bmatrix}$.

Answer: If you had trouble with this, you may want to consult Example 5 on page 25 of your text.

$$\left[\begin{array}{rr|rr} 5 & 3 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{rr|rr} 0 & 1 & 1 & -1 \\ 5 & 2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{rr|rr} 5 & 2 & 0 & 1 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$$\to \left[\begin{array}{rr|rr} 5 & 0 & -2 & 3 \\ 0 & 1 & 1 & -1 \end{array}\right] \to \left[\begin{array}{rr|rr} 1 & 0 & -2/5 & 3/5 \\ 0 & 1 & 1 & -1 \end{array}\right].$$

So

$$A^{-1} = \begin{bmatrix} -2/5 & 3/5 \\ 1 & -1 \end{bmatrix}.$$