Math 354 Summer 2004

Midterm #2

Monday, 8/9/2004

Name: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

Homework grade	
Quiz grade	
Midterm #1 grade	
Midterm #2 grade	
Projected grade	

1 Find the dual of the following linear programming problem.

$$
\begin{array}{l}
\text{Maximize } z = x - y \\
\text{subject to} \\
\begin{array}{rcrcl}
3x & - & y & \leq & 19 \\
-x & + & 7y & \geq & 10 \\
x & + & y & = & 100 \\
x, & & y & \geq & 0
\end{array}
\end{array}
$$

2 Use the Dual Simplex Method to restore feasibility to the following tableau:

	x	y	u_1	u_2	u_3	
y	0	1	$\frac{3}{10}$	$-\frac{1}{10}$	0	$\frac{24}{5}$
x	1	0	$-\frac{4}{5}$	$\frac{3}{5}$	0	$\frac{36}{5}$
u_3	0	0	$\frac{1}{5}$	$-\frac{3}{5}$	1	$-\frac{1}{5}$
	0	0	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{132}{5}$

3 Consider the following linear programming problem.

$$\begin{array}{l} \text{Maximize } z = 4x_1 + 6x_2 + 2x_3 \\ \text{subject to} \\ \begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & \leq & 10 \\ x_1 & + & 4x_2 & & & \leq & 15 \\ x_1 & & & + & x_3 & \leq & 6 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array} \end{array}$$

Applying the Simplex Method to this problem yields the following final tableau

		4	6	2	0	0	0	
$\underline{c}_B$		x_1	x_2	x_3	u_1	u_2	u_3	
0	u_1	0	0	$\frac{1}{4}$	1	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{7}{4}$
6	x_2	0	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{9}{4}$
4	x_1	1	0	1	0	0	1	6
		0	0	$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{75}{2}$

Suppose now that the problem is changed to "Maximize $z = 3x_1 + 6x_2 + 2x_3$." Find an optimal solution to this new problem. (No points will be given for starting from scratch!)

4 Consider the following integer programming problem

$$
\begin{array}{l}
\text{Maximize } z = x + y \\
\text{subject to} \\
\begin{array}{rcrcl}
13x & + & 5y & \leq & 78 \\
-x & + & y & \leq & 0 \\
x, & & y, & \geq & 0, \text{ integral}
\end{array}
\end{array}
$$

Solve this problem using the Cutting Plane Method. (No points will be given for any other methods!)

To help you out, here is the final tableau for the corresponding non-integer linear programming problem (that is, if we ignore the word "integral"):

	x	y	u_1	u_2	
x	1	0	$\frac{1}{18}$	$-\frac{5}{18}$	$\frac{13}{3}$
y	0	1	$\frac{1}{18}$	$\frac{13}{18}$	$\frac{13}{3}$
	0	0	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{26}{3}$

5 Solve the following integer programming problem using the Branch and Bound Method. (No points will be given for any other method!)

$$\begin{array}{l} \text{Maximize } z = 2x + 3y \\ \text{subject to} \\ \begin{array}{rcrcl} 2x & - & y & \leq & 0 \\ & & 2y & \leq & 21 \\ x, & & y, & \geq & 0, \text{ integral} \end{array} \end{array}$$

To help you out, here is the final tableau for the corresponding non-integer linear programming problem (that is, if we ignore the word "integral"):

	x	y	u_1	u_2	
x	1	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{21}{4}$
y	0	1	0	$\frac{1}{2}$	$\frac{21}{2}$
	0	0	1	2	42

6 Consider the following scenario. Model it as an integer programming problem. Be sure to state explicitly what each of your decision variables $x_1, x_2, \ldots$ represent. Do **not** attempt to solve the problem.

The Rutgers football program is attempting to create their 2006 schedule. They are allowed to play up to four non-conference games, which must be chosen from the list below. Also, they can not play the same opponent twice.

Opponent	**Chance of Winning**	**Revenue (in thousands of $)**
Army	60%	$400
Buffalo	65%	$100
Kent State	60%	$100
New Hampshire	70%	$80
Notre Dame	20%	$1,000
Michigan State	25%	$500
Missouri	20%	$450
Ohio State	15%	$850
Navy	50%	$500
Villanova	55%	$100

The Scarlet Knights, despite winning the first intercollegiate football game ever played (against Princeton in 1869 by the score of 6-4, only shortly after Princeton beat Rutgers 40-2 in baseball), have played in only one bowl game, the Garden State Bowl in 1978. To give you an idea of how bad they have been doing, Southern Methodist University has been to four bowl games since 1978.

To give Rutgers a shot to break their bowl-drought, they need a schedule for which they can expect at least two victories against their non-conference opponents. (By a principle called Linearity of Expectation, the number of games they can expect to win is the sum of the probabilities of winning each game individually, so for example, if they scheduled Army, Notre Dame, Missouri, and Ohio State, they could expect to win only $.60 + .20 + .20 + .15 = 1.15$ games.) Beyond this, they would like to maximize revenue.

7 Consider the following primal problem

$$\begin{array}{l} \text{Maximize } z = x_1 + 4x_2 + 5x_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} -x_1 & + & x_2 & + & x_3 & \leq & 4 \\ 3x_1 & + & x_2 & + & x_3 & \leq & 16 \\ & & x_2 & & & \geq & 1 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array} \end{array}$$

and its dual

$$\begin{array}{l} \text{Minimize } z = 4w_1 + 16w_2 - w_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} -w_1 & + & 3w_2 & & & \geq & 1 \\ w_1 & + & w_2 & - & w_3 & \geq & 4 \\ w_1 & + & w_2 & & & \geq & 5 \\ w_1, & & w_2, & & w_3 & \geq & 0 \end{array} \end{array}$$

Without using the Simplex Method on either problem, find optimal solutions to the two problems from the following list. Explain your reasoning.

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 0 \\ 7 \end{bmatrix} \quad \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ 15 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 3 \\ -1 \\ 8 \end{bmatrix} \quad \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ 1 \end{bmatrix} \quad \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ 0 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 12 \\ 7 \end{bmatrix}$$