

## Homework #5

Please write your answers on a separate sheet of paper, and include at least some intermediate steps and English words.

This homework covers sections 3.1-3.4.

1 Consider the linear programming problem

$$\begin{aligned}
 &\text{Maximize } z = 5x_1 + 10x_2 \\
 &\text{subject to} \\
 &\quad x_1 + 3x_2 \leq 50 \\
 &\quad 4x_1 + 2x_2 \leq 60 \\
 &\quad x_1 \leq 5 \\
 &\quad x_1, \quad x_2 \geq 0
 \end{aligned}$$

- (a) State the dual of the preceding LPP.
- (b) Given that (5, 15) is an optimal solution to this LPP, use the Duality Theorem and the principle of complementary slackness to find optimal solution to the dual.

2 The tableau below represents a solution to a linear programming problem that satisfies the optimality criterion, but is infeasible. Use the dual simplex method to restore feasibility.

	5	6	0	0	0	0	
$\underline{c}_B$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$\underline{x}_B$
5 $x_1$	1	0	0	-1	1	0	4
6 $x_2$	0	1	0	1	$-\frac{2}{3}$	0	$\frac{10}{3}$
0 $x_3$	0	0	1	7	-8	0	2
0 $x_6$	0	0	0	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$
	0	0	0	1	1	0	40

3 In class on Monday we divided linear programming problems into three classes:

- (A) No feasible solutions
- (B) Feasible solutions, finite optimal objective value
- (C) Feasible solutions, arbitrarily large objective values

The two-phase simplex method can solve all the problems of type (B).

What would happen if I made a typo on the final exam and asked you to use the two-phase method on a problem of type (A), that is, a problem with no feasible solutions? How far could you get?