Math 354 Summer 2004

Review problems for the Final Exam

Most importantly: Do Homework #7. Some version of each one of those problems will probably show up on the final.

Then: Review the first two midterms. Several questions on the final will be like problems from those.

But also: Here are three problems that are a little different from problems that have been on the exams. Problems like these could also appear on the final. I'll post solutions on Saturday.

1 Consider (again) the following linear programming problem from Midterm #2.

$$\begin{array}{l} \text{Maximize } z = 4x_1 + 6x_2 + 2x_3 \\ \text{subject to} \\ \begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & \leq & 10 \\ x_1 & + & 4x_2 & & & \leq & 15 \\ x_1 & & & + & x_3 & \leq & 6 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array} \end{array}$$

Applying the Simplex Method to this problem yields the following final tableau

		4	6	2	0	0	0	
$\underline{c}_B$		x_1	x_2	x_3	u_1	u_2	u_3	
0	u_1	0	0	$\frac{1}{4}$	1	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{7}{4}$
6	x_2	0	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{9}{4}$
4	x_1	1	0	1	0	0	1	6
		0	0	$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{75}{2}$

This tableau represents the optimal solution

$$\underline{x} = \begin{bmatrix} 6 \\ \frac{9}{4} \\ 0 \end{bmatrix}$$

Suppose now that the problem is changed to "Maximize $z = (4 + \Delta)x_1 + 6x_2 + 2x_3$" for some real number Δ. For what range of Δ is

$$\underline{x} = \begin{bmatrix} 6 \\ \frac{9}{4} \\ 0 \end{bmatrix}$$

still an optimal solution for the new problem? (Hint: you may want to try a specific value or two for Δ first, to get a feel for what I'm asking.)

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2 Given that

$$\underline{x} = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

is a solution to the primal problem

$$\begin{array}{l} \text{Maximize } z = x_1 + 4x_2 + 5x_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} -x_1 & + & x_2 & + & x_3 & \leq & 4 \\ 3x_1 & + & x_2 & + & x_3 & \leq & 16 \\ & & x_2 & & & \geq & 1 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array} \end{array}$$

use the Principle of Complementary Slackness to find a solution to the dual problem

$$\begin{array}{l} \text{Minimize } z = 4w_1 + 16w_2 - w_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} -w_1 & + & 3w_2 & & & \geq & 1 \\ w_1 & + & w_2 & - & w_3 & \geq & 4 \\ w_1 & + & w_2 & & & \geq & 5 \\ w_1, & & w_2, & & w_3 & \geq & 0 \end{array} \end{array}$$

3 Consider the following linear programming problem.

$$\begin{array}{l} \text{Maximize } z = 3x_1 - x_2 + 2x_3 + 4x_4 \\ \text{subject to} \\ \begin{array}{rcrcrcrcl} & & x_2 & + & 7x_3 & + & 2x_4 & \geq & 3, \\ x_1 & + & 2x_2 & + & x_3 & & & = & 9, \\ 2x_1 & + & 3x_2 & + & x_3 & - & 4x_4 & \leq & 7, \end{array} \\ x_j \geq 0,\ j = 1, 2, 3, 4. \end{array}$$

When solving it, the following tableau came up

	x_1	x_2	x_3	x_4	u_1	u_2	
x_4	0	$\frac{1}{4}$	$\frac{1}{4}$	1	0	$-\frac{1}{4}$	$\frac{11}{4}$
u_1	0	$-\frac{1}{2}$	$-\frac{13}{2}$	0	1	$-\frac{1}{2}$	$\frac{5}{2}$
x_1	1	2	1	0	0	0	9
	0	8	2	0	0	16	38

Could it be correct? Explain.