Quiz #8 for Math 135 3/31/2004

ANSWER SHEET

1. The derivative of a certain function $f(x)$ is given by $f'(x) = x(x^2-4)(x-3)^4$. Find the local minimum(s) of f and find where f is decreasing.

Answer: The critical numbers are where $f'(x) = 0$, so in this case they are just $0, -2, 2$, and 3. Now we make a chart and check the sign of $f'(x)$ for values between the critical numbers:

$$\begin{aligned} f'(-3) &= -19440 \\ f'(-1) &= 768 \\ f'(1) &= -48 \\ f'(2.5) &= 0.3515625 \\ f'(4) &= 48. \end{aligned}$$

Therefore f is decreasing on $(-\infty, -2)$ and $(0, 2)$, and it has local minima at $x = -2$ and $x = 2$.

2. Find the second-order critical numbers of $f(x) = \frac{x^4}{4} - x^3 - \frac{9x^2}{2}$ and tell where the graph is concave up and concave down.

Answer: To find the second-order critical numbers of f we find the secdond derivative, $f''(x)$, and set it equal to 0:

$$f'(x) = x^3 - 3x^2 - 9x,$$

so

$$f''(x) = 3x^2 - 6x - 9 = 3(x+1)(x-3),$$

and thus the second-order critical numbers are -1 and 3.
The graph of $f(x)$ is concave up when $f''(x)$ is positive and concave down when $f''(x)$ is negative, so we want to test $f''(x)$ at three points: one value less than -1, another value between -1 and 3, and a final value greater than 3:

$$\begin{aligned} f''(-2) &= 15 \\ f''(0) &= -9 \\ f''(4) &= 15. \end{aligned}$$

This means that f is concave up on $(-\infty, -1)$ and $(3, \infty)$ and concave down on $(-1, 3)$.