Quiz #7 for Math 135 3/24/2004

ANSWER SHEET

1. The radius of a spherical ball is measured as 14 in., with possible error of $\frac{1}{8}$ in. Use differentials to estimate the maximum propagated error in computing the volume. (The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

Answer: The change in the volume, ΔV, is approximated by

$$\Delta V \approx V'(r)\Delta r.$$

Also, $V'(r) = 4\pi r^2$, and $\Delta r \leq \frac{1}{8}$, so

$$\Delta V \leq 4\pi(14)^2\frac{1}{8} = 98\pi.$$

2. Find the absolute minimum and maximum of $f(u) = 1 - u^{2/3}$ on the interval $[-1, 1]$.

Answer: We start by taking the derivative of f:

$$\begin{aligned} f'(u) &= -\frac{2}{3}u^{-1/3} \\ &= \frac{-2}{3u^{1/3}} \end{aligned}$$

Can that ever be 0? No, no matter what we divide -2 by, we'll never get 0.

Now we have to check where f' does not exist. That happens at $u = 0$, because $0^{1/3} = 0$, and we can't divide by 0.

So our only critical number is 0, and we just have to evaluate f at 0 and the two endpoints, -1 and 1:

$$\begin{aligned} f(0) &= 1 - 0^{2/3} = 1 \\ f(-1) &= 1 - (-1)^{2/3} = 1 - 1 = 0 \\ f(1) &= 1 - 1^{2/3} = 1 - 1 = 0 \end{aligned}$$

(By the way, how so many of you got $(-1)^{2/3} = -1$ is a mystery to me.)

This gives our final answer: f achieves its minimum (0) at $u = 1$ and $u = -1$ and it achieves its maximum (1) at $u = 0$.