Quiz #6 for Math 135 3/10/2004

ANSWER SHEET

1. Find $\frac{dy}{dx}$ by implicit differentiation: $xy(2x+3y)=2$.

Answer: I would start by expanding the equation: $2x^2y+3xy^2=2$. Now differentiate both sides. To differentiate $2x^2y$ and $3xy^2$ we are going to have to use the product rule:

$$\frac{d}{dx}(2x^2)\cdot y+(2x^2)\cdot\frac{d}{dx}(y)+\frac{d}{dx}(3x)\cdot y^2+(3x)\cdot\frac{d}{dx}(y^2)=0$$
$$4xy+2x^2\frac{dy}{dx}+3y^2+3x\cdot 2y\frac{dy}{dx}=0.$$

Now we have to solve for $\frac{dy}{dx}$:

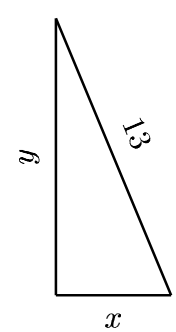
$$2x^2\frac{dy}{dx}+3x\cdot 2y\frac{dy}{dx}=-4xy-3y^2,$$

so

$$\frac{dy}{dx}=\frac{-4xy-3y^2}{2x^2+6xy}.$$

2. A ladder 13 ft long rests against a vertical wall and is sliding down the wall at the rate of 3 ft/s at the instant the foot of the latter is 5 ft from the base of the wall. At this instant, how fast is the foot of the ladder moving away from the wall?

Answer: First draw a picture:



We start with the Pythagorean Theorem, which says that $x^2+y^2=13^2$. If we differentiate this with respect to time we get

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0.$$

We want to find the rate at which the foot of the ladder is moving away from the wall, which is $\frac{dx}{dt}$.

We are given that the ladder is "sliding down the wall at the rate of 3 ft/s at the instant the foot of the latter is 5 ft from the base of the wall," so when $x = 5$, $\frac{dy}{dt} = -3$. That's two of the three things we need to know; the other one is y. To get y we use the Pythagorean Theorem again:

$$5^2 + y^2 = 13^2,$$

so $y = 12$. This gives us

$$2 \cdot 5 \cdot \frac{dx}{dy} + 2 \cdot 12 \cdot (-3) = 0,$$

so $\frac{dx}{dt} = 72/10 = 7.2$.