Quiz #3 for Math 135 2/11/2004

ANSWER SHEET

1. Find constants a and b so that

$$f(x) = \begin{cases} \frac{ax-4}{x-2} & \text{if } x \neq 2 \\ \\ b & \text{if } x = 2 \end{cases}$$

is continuous.

Answer: The only "suspicious point" is $x = 2$, since everywhere else f will be continuous. The first problem to deal with is to find a value of a so that $\lim_{x\to 2} \frac{ax-4}{x-2}$ exists.

Since the denominator of $\frac{ax-4}{x-2}$ is 0 at $x = 2$, our only chance to make the limit exist is to make the numerator 0 at $x = 2$ as well. So we want $a(2) - 4 = 0$, which means $a = 2$.

Now we just have to make sure that $\lim_{x\to 2} f(x) = f(2)$. Since

$$\lim_{x\to 2} \frac{2x-4}{x-2} = \lim_{x\to 2} 2 = 2,$$

we need to have $f(2) = 2$, and because $f(2) = b$, this means that we should set $b = 2$.

2. A certain bank pays 6% interest compounded continuously. How long will it take for \$825 to double?

Answer: The value of the investment at time t is given by $f(t) = 825e^{0.06t}$. So, we want to find the value of t for which $f(t) = 2 \cdot 825 = 1650$:

$$f(t) = 1650$$

means

$$825e^{0.06t} = 1650$$

means

$$e^{0.06t} = 2$$

means

$$0.06t = \ln 2$$

so the solution is $t = \frac{\ln 2}{0.06} \approx 11.55$.

As an aside, you might have heard of the "Rule of 72" for figuring out how long it takes for your money to double. The rule states that your money doubles in about about 72 divided by the interest rate years, so for our example it would give $72/6 = 12$ years, about a half a year too long. Actually, 72 is the wrong number to use; 69 would be a lot better. Why does the rule work?