Quiz #2 for Math 135 2/4/2004

ANSWER SHEET

First, a general comment: unless specifically requested, you are supposed to find limits algebraically, *not* using a table of values or your calculator.

1. Evaluate $\lim_{x\to 0} \frac{(x+1)^2-1}{x}$.

Answer: The idea is to expand the $(x+1)^2$ in the numerator, and then we will be able to simplify the fraction and get the limit. A common mistake (although this time I was probably nicer about it than I should have been) was to drop the limits signs, for example, writing $\lim_{x\to 0} \frac{(x+1)^2-1}{x} = \frac{x^2+2x}{x}$. That's not true! If you want to take what's inside the limit and simplify it before returning to the limit, that's fine, but otherwise, you have to keep the lim around until you actually take the limit. Here's the solution:

$$\begin{aligned} \lim_{x\to 0} \frac{(x+1)^2-1}{x} &= \lim_{x\to 0} \frac{x^2+2x+1-1}{x} \\ &= \lim_{x\to 0} \frac{x^2+2x}{x} \\ &= \lim_{x\to 0} x+2 \\ &= 2. \end{aligned}$$

2. Evaluate $\lim_{x\to 0} \frac{\sin 4x}{9x}$.

Answer: Here's what you can't do: you can't pull the 4 out of $\sin 4x$. Remember that $\sin 4x$ is a function applied to $4x$, and $\sin 4x \neq 4\sin x$.

What you want to do instead is get whatever we are taking the sin of below the sin function and then use the fact that $\lim_{u\to 0} \frac{\sin u}{u} = 1$.

$$\begin{aligned} \lim_{x\to 0} \frac{\sin 4x}{9x} &= \lim_{x\to 0} \frac{\sin 4x}{9x} \cdot \left(\frac{9}{4} \cdot \frac{4}{9}\right) \\ &= \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot \frac{4}{9} \\ &= \frac{4}{9} \lim_{x\to 0} \frac{\sin 4x}{4x} \\ &= \frac{4}{9} \cdot 1 \\ &= \frac{4}{9}. \end{aligned}$$