Quiz #11 for Math 135 4/21/2004

ANSWER SHEET

1. Suppose the total cost of manufacturing x units of a certain commodity is

$$C(x) = 3x^2 + x + 48$$

dollars. Determine the minimum average cost.

Answer: One way to do this problem is just to figure out the average cost and minimize it. If we make 5 units then our total cost is $C(5)$ and our average cost is thus $C(5)/5$. Thus is we make x units our total cost will be $C(x)$ and our average cost will be $C(x)/x$. The average cost function is therefore

$$AC(x) = \frac{C(x)}{x} = \frac{3x^2 + x + 48}{x} = 3x + 1 + \frac{48}{x}.$$

To minimize this we take its derivative

$$AC'(x) = 3 - \frac{48}{x^2}$$

and set it equal to 0; $AC'(x) = 0$ means that $3 = \frac{48}{x^2}$, so $x = \pm 4$. We can rule out the $x = -4$ critical number, so the minimum must occur when $x = 4$.

Finally, we have to find the minimum average cost, which is the value of the AC function when we plug in $x = 4$:

$$AC(4) = 3(4) + 1 + \frac{48}{4} = 25.$$

2. Suppose $f(x)$ satisfies $f'(x) = x^2 + 3x$ and the graph of $f(x)$ contains the point $(0, 0)$. What is $f(x)$?

Answer: We know that

$$f(x) = \int x^2 + 3x \; dx = \frac{x^3}{3} + \frac{3x^2}{2} + C,$$

so we just have to find C.

Since they gave us that the graph of $f(x)$ contains $(0, 0)$, we know that $f(0) = 0$ and we plug that in to the above:

$$f(0) = \frac{0^3}{3} + \frac{3(0)^2}{2} + C = C = 0,$$

so $C = 0$.

The answer is therefore $f(x) = \frac{x^3}{3} + \frac{3x^2}{2}$.