

ANSWER SHEET

1. A poster is to contain 108 cm^2 of printed matter, with margins of 6 cm each at top and bottom and 2 cm on the sides. What is the minimum cost of the poster if it is to be made of material costing 20 cents/ cm^2 .

Answer: Let x be the width in cm of the printed material on the poster and y the height. Since we are supposed to have 108 cm^2 of printer material,

$$xy = 108.$$

With the margins, the total area of the poster will be

$$A = (x + 4)(y + 12),$$

and since we are paying 20 cents/ cm^2 for the poster, the cost is

$$C = .20(x + 4)(y + 12),$$

and this is what we want to minimize.

First solve the equation $xy = 108$ in terms of y : $y = 108/x$ (or solve it in terms of x if you like). Then substitute this into C :

$$C = .20(x + 4) \left(\frac{108}{x} + 12 \right) = 31.20 + 2.40x + \frac{86.40}{x}.$$

We want to minimize C so we take a derivative:

$$C' = 2.40 - \frac{86.40}{x^2}.$$

Now we want to find the critical numbers of C so we set C' equal to 0. If $C' = 0$ then $\frac{86.40}{x^2} = 2.40$, so

$$x = \pm 6.$$

We can rule out the solution $x = -6$, and since the only remaining critical number is $x = 6$, we can be assured that that value minimizes C .

Finally, we have to give the minimum cost, which is the value of C when we plug in $x = 6$:

$$.20(6 + 4) \left(\frac{108}{6} + 12 \right) = 60.$$