A few review problems for Math 152

Solutions are available on the class webpage at `http://math.rutgers.edu/~vatter/`.

1 Let $\mathcal{R}$ denote the region bounded by $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$.

(a) Find the volume of the solid that results from rotating $\mathcal{R}$ around the x-axis.

(b) Find the volume of the solid that results from rotating $\mathcal{R}$ around the y-axis.

2 Determine whether each of the following improper integrals converge or diverge. Calculate the value of any convergent integral.

$$\int_0^\infty xe^{-3x^2}\,dx \qquad\qquad \int_{-2}^0 \frac{dx}{2+x}$$

3 Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{y^2(x^2+2x+3)}{(x^2+1)(x+1)}$$

satisfying the initial condition $y(0) = 2$. In the answer express y as a function of x.

4 Let $f(x) = \dfrac{3x}{2+5x^4}$.

(a) Find the power series representation for $f(x)$.

(b) What is the radius of convergence of this series?

5 Let k be a fixed integer less than 8. Show that

$$\int_0^{1/\sqrt{2}} \frac{x^{k-1}}{1-x^8}\,dx = \frac{1}{2^{k/2}}\sum_{n=0}^{\infty}\frac{1}{16^n(8n+k)}.$$

6 Show that

$$\int_0^{1\sqrt{2}} \frac{4\sqrt{2}-8x^3-4\sqrt{2}x^4-8x^5}{1-x^8} = \pi.$$

Hint: Once we substitute $u = \sqrt{2}x$, the integral becomes

$$\int_0^1 \frac{16u-16}{u^4-2u^3+4u-4}\,du,$$

and using partial fractions, this is equal to

$$\int_0^1 \frac{4u}{u^2-2}\,du - \int_0^1 \frac{4u-8}{u^2-2u+2}\,du.$$

7 Using the previous two problems, show that

$$\sum_{n=0}^{\infty} \frac{1}{16^n} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) = \pi,$$

thereby proving the famous Bailey-Borwein-Plouffe formula for π, discovered in 1995. (Therefore, if you had done this problem ten years ago, you would be famous.)[1]

[1] To see how the formula was actually discovered, read their original paper, "On the rapid computation of various polylogarithmic constants," *Mathematics of Computation* **66** (1997), 903–913 or the book *Mathematics by Experiment* by Borwein and Bailey