Math 152 Midterm 2 Review Sheet

1 Determine whether $\lim_{n\to\infty} a_n$ exists when the sequence $\{a_n\}$ is defined as follows. Find the limit if it exists.

$$a_n = \frac{(n+2)!}{n!(3+5n)^2} \qquad a_n = \frac{5n}{\ln(2+3e^n)} \qquad a_n = (2n)^{1/n}$$

2 Determine whether the following series converge. Sum the series that do converge.

$$\sum_{n=1}^{\infty} \frac{3^n + 4^n}{5^n} \qquad \sum_{n=1}^{\infty} \left(1 + \frac{(-1)^n}{n^2}\right) \qquad \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$$

3 For which values of p does $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge? For which values of p does $\sum_{n=1}^{\infty} \frac{1}{p^n}$ converge?

4 Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=0}^{\infty} \frac{n^3}{3^n} \qquad \sum_{n=1}^{\infty} \frac{\sin n}{n^2} \qquad \sum_{n=1}^{\infty} \frac{n^3}{n^4+1} \qquad \sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^4+1}$$

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}} \qquad \sum_{n=0}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}} \qquad \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

5 We wish to approximate the sum S of the series $\sum_{n=1}^{\infty} a_n$ with the partial sum $s_N = \sum_{n=1}^{N} a_n$. How large should N be to ensure that s_N approximates S with an error less that 10^{-4} for each of the following series?

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

6 Find the radius of convergence of the following power series and determine the convergence behavior at the endpoints of the interval of convergence. Show your reasoning.

$$\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n} \qquad \sum_{n=1}^{\infty} \frac{5^n x^n}{n^{1/4}} \qquad \sum_{n=1}^{\infty} \frac{n^2}{(3n+1)!} x^n$$

7 By manipulating series that you already know, find Maclaurin series for each of the following functions.

$$\frac{1}{1+x^3} \qquad x\sin 2x \qquad \frac{\ln(1+x^2)}{x}$$

$$\frac{e^x - e^{-x}}{2} \qquad e^{3x^2} \qquad -\left(\frac{1}{1+x}\right)^2$$

8 Find the first few terms of the Maclaurin series for the following functions.

$$(\ln(1+x))^2 \qquad \frac{\cos x}{1-x} \qquad \sin^2 x + \cos^2 x$$

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9 Use the Maclaurin series for e^x to obtain a power series for the function

$$f(x) = \int_0^x \frac{1 - e^{t^2}}{t}\, dt.$$

10 Find $\lim_{x \to 0} \dfrac{(\sin x - x)^2}{(\cos 5x - 1)^3}$.

11 Find the Maclaurin series for $\arctan x$ and use this to write π as the sum of an infinite series.