

Workshop and homework

11.6 1,11,19,22
Homework: 11.7 9,14,35
 11.8 3,7,30

1 The **Alternating Series Test** says that the alternating series

$$\sum_{n=1}^{\infty} (-1)^n b_n = -b_1 + b_2 - b_3 + b_4 - b_5 + \dots$$

converges if the following two things are true:

- (i) $b_{n+1} \leq b_n$ (the terms are getting smaller), and
- (ii) $\lim_{n \rightarrow \infty} b_n = 0$ (the terms tend to 0).

Important: If either of these two conditions do not hold, then the Alternating Series Test says nothing about your series.

Which of the following series satisfy the conditions of the Alternating Series Test?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n!)^2} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \quad \sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{n^6 + 1}$$

2 The **Ratio Test** says that if the limit $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ exists and is less than 1, then the series $\sum a_n$ is absolutely convergent. On the other hand, if this limit exists and is greater than 1, then the series $\sum a_n$ diverges. If this limit is exactly 1, then the Ratio Test says nothing about $\sum a_n$.

In the following series x is a real number. Use the ratio test to determine all values of x for which the series converges absolutely.

$$\sum_{n=1}^{\infty} \frac{nx^n}{n^2 + 1} \quad \sum_{n=1}^{\infty} \frac{n^2(x-1)^n}{2^n} \quad \sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$