

Math 152, Fall 2003 Final Exam Review Problems

For answers, check <http://math.rutgers.edu/~vatter/> on or after Sunday (12/7).

1.

- Sketch the region R bounded by the curves $y = x^2 + 1$ and $y = 3 - x^2$.
- Find the area of R .
- Compute the volume which results when the region R is revolved about the x -axis.

2. Integrate the following:

- $\int \frac{15x}{(x-4)(x^2+4)} dx$
- $\int \frac{7}{2x\sqrt{x^2+9}} dx$
- $\int \sin^4 x \cos^5 x dx$
- $\int x^2 e^{-7x} dx$
- $\int \frac{(\ln x)^2}{x} dx$
- $\int x^2 \ln x dx$

3. Solve the initial value problem

$$\frac{dy}{dx} = \frac{-3 \sin 3x}{y}, \quad y(0) = 1$$

explicitly as a function of x .

4. Find the length of the curve given by

$$x = e^{2t} - 2t, \quad y = 4e^t$$

between $1 \leq t \leq 3$.

5.

- Sketch the two curves $r = 1 + 2 \sin \theta$ and $r = 4 \sin \theta$, $0 \leq \theta \leq \pi$, labelling the x and y -coordinates of the points of intersection.
- Set up but DO NOT EVALUATE an integral representing the area of the region outside $r = 1 + 2 \sin \theta$ but inside $r = 4 \sin \theta$.

6. For each of the following series, tell whether they are absolutely convergent, conditionally convergent, or divergent.

- $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^2 + 3}$

- b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^2}$
 c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^{1/n}}$
 d) $\sum_{n=34}^{\infty} \frac{1+e^n}{7^n}$
 e) $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

7. How many terms of the series

$$\sum_{n=1}^{\infty} \frac{4}{n^{5/2}}$$

are required to approximate the sum of the series to within 0.001?

8. Find the interval of convergence for each of the following series

- a) $\sum_{n=1}^{\infty} \frac{(3x)^n}{n+4}$
 b) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$
 c) $\sum_{n=0}^{\infty} \frac{n^2 x^n}{(3n+1)!}$
 d) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n2^n}$

9. Use the Maclaurin series for e^x to get an infinite series for the integral

$$I = \int_0^1 e^{-x^2/2} dx.$$