Assignment for First Two Class Meetings.

Integration is far more difficult than differentiation. So to be good at integration, one should practice on it far more than on differentiation. Just inside the hard cover at the end of our textbook, there are 120 integral formulas. Of course, we do not need to memorize all of these. We can look up the less often used formulas as needed. But there is a short list of formulas used so often that the scientist simply could not afford to stop the normal flow of study to find another formula. Furthermore, a well-chosen short list must be memorized. For this list is used as a foundation to organize the overall picture of the whole integration problem. So if one does not know the basic list, then he or she would be dumbfounded as to the type of formula to look for to solve a given integral problem.

At the beginning of the semester, each student is given a printed list of these basic 16 integral formulas along with some common trigonometric identities. But in case the student misplaces his or her copy, the student may print out another copy from my web page. It would be a good idea for the student to keep this list of the basic 16 integration formulas and trig identities in a special notebook for the calculus course.

It is essentially impossible to be good at integration if one does not memorize the basic 16 integral formulas. This is very easy to do if we learn them in small groups as follows:

B1–B3 first: then B14–B16; then B4–B9; B10–B13 last.

In 1–18, find each integral simply by applying a basic integral formula without making a substitution. In order to maximize your personal growth and strength in integration, I strongly advise you not to make a substitution in these problems. To continue to substitute in these integrals, which actually **fit a basic integral formula**, will surely impede your progress in developing good skills for integration.

In introducing some students to integration, it may be helpful to let them use the substitution method in the beginning with the hope that soon they will abandon the "training wheels phase" of their learning process in integration; however, it is often difficult to give up bad habits!

As you begin this set of exercises, you should first look over B1–B3 and try to commit them to memory; this mission should absolutely be completed by the time you finish Ex 1–12.

1. **(a)** $\int x^2\left(7+x^3\right)^{\frac{5}{2}}\ dx = \int \left(7+x^3\right)^{\frac{5}{2}}\frac{1}{3}(3x^2\,dx)$

$= \frac{1}{3}\int u^{\frac{5}{2}}du = \frac{1}{3}\frac{2}{7}u^{\frac{7}{2}} + C = \frac{2}{21}\left(7+x^3\right)^{\frac{7}{2}} + C$

(b) Notice that we should shorten the solution in Part (a) as follows:

$\int x^2(7+x^3)^{\frac{5}{2}}dx = \frac{1}{3}\int(7+x^3)^{\frac{5}{2}}3x^2\,dx =$

$\frac{1}{3}\frac{2}{7}(7+x^3)^{\frac{7}{2}} + C$ by B1.

In 2–18, you should try using the format in 1(b)– even if, at first, you may not quite like it as much as the old substitution method sometimes suggested for beginners in integration. It is hoped that **very soon**, you will appreciate **some** of the advantages of the new format. Quite soon, you should see more advantages of the new format and like it much,much more than the old substitution method. Down the road, we will use the substitution method for some integrals which cannot quickly be seen to fit one of the basic 16 integral formulas.

2. $\int \frac{x\ dx}{\sqrt{7+3x^2}} = \frac{1}{6}\int(7+3x^2)^{-\frac{1}{2}}6x\,dx = \frac{1}{6}\frac{2}{1}(7+3x^2)^{\frac{1}{2}} + C$ by B1

3. $\int x\sqrt[3]{7-4x^2}\,dx = -\frac{1}{8}\int(7-4x^2)^{\frac{1}{3}}(-8x\,dx) =?$ by B1

4. $\int \cos 3x\ \sqrt{7+\sin 3x}\,dx = \frac{1}{3}\int(7+\sin 3x)^{\frac{1}{2}}(\cos 3x\ \ 3\,dx) =?$ by B1

$= \frac{1}{3}\frac{2}{3}(7+\sin 3x)^{\frac{3}{2}} + C$

5. $\int e^{2x}\sqrt[3]{7+4e^{2x}}\,dx = \frac{1}{8}\int(7+4e^{2x})^{\frac{1}{3}}\ (4e^{2x}\ \ 2\,dx) = \frac{1}{8}\frac{3}{4}(7+4e^{2x})^{\frac{4}{3}} + C$ by B1

6. $\int \frac{\sqrt{7+3\ln x}}{x}dx = \int(7+3\ln x)^{\frac{1}{2}}\ \frac{1}{x}\ dx = \frac{1}{3}\int(7+3\ln x)^{\frac{1}{2}}\ \frac{3}{x}\,dx$

$=?$ by B1 $\qquad \frac{2}{9}(7+3\ln x)^{\frac{3}{2}} + C$

7. $\int \sin 3x\ \cos 3x\ dx \quad = \frac{1}{3}\int(\sin 3x)^1(\cos 3x\ 3dx) =?$ by B1

8. **(a)** $\displaystyle\int \frac{x\ dx}{7+3x^2} = \frac{1}{6} \int \frac{6x\ dx}{7+3x^2} = \frac{1}{6}\ \ln|7+3x^2| + C$ by B2

$\displaystyle= \frac{1}{6}\ \ln(7+3x^2) + C$

(b) $\displaystyle\int \frac{x^2\ dx}{7+x^3} = () \int \frac{()x^2 dx}{7+x^3}$ Fill blanks.

$\displaystyle= \frac{1}{3} \ln|7+x^3| + C$ by B2

9. **(a)** $\displaystyle\int \frac{\cos 3x\ dx}{7+\sin 3x} = \cdots \quad = \frac{1}{3}\ln(7+\sin 3x) + C$ by B2

(b) $\displaystyle\int \frac{\cos 3x\ dx}{(7+\sin 3x)^2} = \int (7+\sin 3x)^{-2}\ \cos 3x\ dx$

$\displaystyle= \cdots \quad = -\ \frac{1}{3(7+\sin 3x)} + C$ by B1

10. **(a)** $\displaystyle\int \frac{dx}{x(7+3\ln x)} = \int \frac{\frac{1}{x}\ dx}{7+3\ln x} = (\ )\ \int \frac{\frac{(\ )}{x}\ dx}{7+3\ln x}$

$= ?$ by B2 $\qquad \displaystyle\frac{1}{3}\ \ln|7+3\ln x| + C$

(b) $\displaystyle\int \frac{(\ln x)^3}{x}\ dx = \int (\ln x)^3\ \frac{1}{x}\ dx = ?$ by B1

(c) $\displaystyle\int \frac{\ln x}{x}\ dx = \int (\ln x)^1\ \frac{1}{x}\ dx = ?$ by B1

(d) $\displaystyle\int \frac{1}{x\ln x}\ dx = \int \frac{\frac{1}{x}\, dx}{\ln x} = ?$ by B2

11. **(a)** $\displaystyle\int x^2\, e^{2x^3}\ dx = (\ ) \int e^{2x^3}\ (\ )x^2 dx = ?$ by B3

$\displaystyle\frac{1}{6}\ e^{2x^3} + C$

(b) $\displaystyle\int (\sin 3x)\ e^{\cos 3x}\ dx = -\frac{1}{3}\ \int e^{\cos 3x}\ (-\sin 3x\ 3dx) = ?$ by B3

$\displaystyle-\frac{1}{3}\ e^{\cos 3x} + C$

(c) To do this part, we should recall that $u = e^{\ln u}$ & $u^v = e^{v\ln u}$ if $u > 0$.

$\displaystyle\int x\, 3^{x^2} dx = (\ ) \int e^{x^2 \ln 3}(\ )x\, dx = ?$ by B3

$\displaystyle= \frac{1}{2\ln 3}\ e^{x^2\ln 3} + C \quad = \quad \frac{3^{x^2}}{2\ln 3} + C$

12. **(a)** $\displaystyle\int (7+e^x)^{\frac{3}{2}}\ e^x dx = ?$ by B1 $\qquad \displaystyle\frac{2}{5}(7+e^x)^{\frac{5}{2}} + C$

(b) $\int \frac{(7+\ln x)^{\frac{3}{2}}}{x}\, dx = ?$ $\qquad \frac{2}{5}(7+\ln x)^{\frac{5}{2}} + C$

(c) $\int \frac{e^x dx}{7+e^x} = ?$ $\qquad \ln(7+e^x) + C$

(d) $\int \frac{e^{-x} dx}{7+e^{-x}}$ $\qquad -\ln(7+e^{-x}) + C$

(e) $\int \frac{dx}{7+e^x} = \int \frac{e^{-x} dx}{7e^{-x}+1}$ $\qquad -\frac{1}{7}\ln(7e^{-x}+1) + C$

(f) $\int \frac{\ln x}{x} dx$ $\qquad \frac{1}{2}(\ln x)^2 + C$

(g) $\int \frac{dx}{x \ln x} = \int \frac{\frac{1}{x} dx}{\ln x}$ $\qquad \ln|\ln x| + C$

At this point, we should look over B14–B16 and commit these formulas to memory.

13(a) $\int \frac{x\ dx}{9+x^4} = \int \frac{x\ dx}{3^2+(x^2)^2} = \frac{1}{2}\int \frac{2x\ dx}{3^2+(x^2)^2} = \frac{1}{2}\frac{1}{3} \arctan \frac{x^2}{3} + C$ by B15

13(b) $\int \frac{x^3 dx}{9+x^4} = \frac{1}{4}\int \frac{4x^3 dx}{9+x^4} = \frac{1}{4}\ln(9+x^4) + C$ by B2

14(a) $\int \frac{e^{3x} dx}{9+e^{6x}} = \frac{1}{3}\int \frac{e^{3x}\, 3\, dx}{3^2+(e^{3x})^2}$

$= \frac{1}{3}\,\frac{1}{3} \arctan \frac{e^{3x}}{3} + C$ by B15

14(b) $\int \frac{e^{6x} dx}{9+e^{6x}} = (\ )\int \frac{e^{6x}(\ ) dx}{9+e^{6x}} = \frac{1}{6}\ln(9+e^{6x}) + C$ by B2

15(a) $\int \frac{x^3 dx}{\sqrt{9-x^8}} = (\ )\int \frac{(\ )x^3 dx}{\sqrt{3^2-(x^4)^2}} = \frac{1}{4} \arcsin \frac{x^4}{3} + C$ by B14

15(b) $\int \frac{e^{4x} dx}{\sqrt{9-e^{8x}}} = (\ )\int \frac{e^{4x}(\ ) dx}{\sqrt{3^2-(e^{4x})^2}} = \frac{1}{4} \arcsin \frac{e^{4x}}{3} + C$ by B14

15(c) $\int \frac{e^{8x} dx}{\sqrt{9-e^{8x}}} = \cdots = -\frac{1}{8}\frac{2}{1}(9-e^{8x})^{\frac{1}{2}} + C$ by B1

16(a) $\int \frac{dx}{x\sqrt{9x^2-25}} = \int \frac{3\ dx}{(3x)\sqrt{(3x)^2-5^2}} = \frac{1}{5} \operatorname{arcsec} \frac{3x}{5} + C$ by B16

16(b) $\displaystyle\int \frac{dx}{x\sqrt{9x^4-25}} = \int \frac{3x\ dx}{(3x^2)\sqrt{(3x^2)^2-5^2}} = \frac{1}{2}\int \frac{6x\ dx}{3x^2\sqrt{(3x^2)^2-5^2}}$

$\displaystyle = \frac{1}{2}\frac{1}{5}\ \text{arcsec}\ \frac{3x^2}{5} + C$ by B16

16(c) $\displaystyle\int \frac{dx}{\sqrt{e^{4x}-25}} = \int \frac{e^{2x}dx}{e^{2x}\sqrt{(e^{2x})^2-5^2}}$

$\displaystyle = (\ )\int \frac{e^{2x}(\ )dx}{e^{2x}\sqrt{(e^{2x})^2-5^2}} = \frac{1}{2}\frac{1}{5}\ \text{arcsec}\ \frac{e^{2x}}{5} + C$ by B16

17(a) $\displaystyle\int \frac{dx}{x[1+(\ln x)^2]} = \int \frac{\frac{1}{x}\ dx}{1^2+(\ln x)^2}$ =? by B15

17(b) $\displaystyle\int \frac{dx}{\sqrt{x}\ (1+x)} = (\ )\int \frac{(\ )\frac{1}{\sqrt{x}}\ dx}{1^2+(\sqrt{x})^2}$ Use B15. $\quad 2\ \arctan\sqrt{x}+C$

18. $\displaystyle\int x^2\, 5^{x^3}\ dx$ See 11(c). $\qquad \displaystyle\frac{5^{x^3}}{3\ \ln 5}+C$

Now we should remember the formula for integration by parts and use it for many of the remaining problems:

$$\int u\, dv \ = \ uv - \int v\, du.$$

19. $\displaystyle\int (x)\, [e^{3x}\ dx]$ $\qquad \displaystyle\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$

20. $\displaystyle\int (x)\, [\cos 3x\ dx]$ $\qquad \displaystyle\frac{1}{3}x\ \sin 3x + \frac{1}{9}\cos 3x + C$

21. $\displaystyle\int (x)\, [\sin 4x\ dx]$ $\qquad \displaystyle -\frac{1}{4}\, x\cos 4x + \frac{1}{16}\ \sin 4x + C$

22. $\displaystyle\int (x)\, [\sec^2 x\ dx]$ $\qquad x\tan x\ +\ \ln|\cos x| + C$ See B10–13.

23. $\displaystyle\int x^2\ \ln x\ dx = \int (\ln x)\, [x^2\, dx]$ $\qquad \displaystyle\frac{x^3}{3}\ \ln x - \frac{x^3}{9} + C$

24. $\int x^6 \ln x \, dx = \int (\ln x)\,[x^6 dx]$ $\qquad\qquad \dfrac{x^7}{7} \ln x - \dfrac{x^7}{49} + C$

25. $\int \sqrt{x} \ln x \, dx = \int (\ln x)\,[x^{\frac{1}{2}} \, dx]$ $\qquad\qquad \dfrac{2}{3} x^{\frac{3}{2}} \ln x - \dfrac{4}{9} x^{\frac{3}{2}} + C$

In general, do not let $dv = \ln x$ since dv should be **easily integrated** to give something **simple.**

26. $\int \ln x \, dx \;=\; \int (\ln x)\; [dx]$ $\qquad\qquad x \; \ln x \;-\; x \;+\; C$

27. Replace x by u in the final answer in #26 to obtain a general formula for $\int \ln u \, du$. $\quad\cdots$ The answer is:

$$\int \ln u \; du \;=\; u \ln u \;-\; u \;+\; C$$

28. Use the general formula in #27 to find the integral.

(a) $\int \ln(2x+1)\; dx = \dfrac{1}{2} \int \ln(2x+1)\; 2\, dx$

$$= \frac{1}{2}[(2x+1)\; \ln(2x+1) \;-\; (2x+1)] + C \text{ by \#27}$$

$$= (x + \frac{1}{2}) \ln(2x+1) - x - \frac{1}{2} + C$$

$$= (x + \frac{1}{2}) \ln(2x+1) - x + C'$$

$$\text{where } C' = -\frac{1}{2} + C$$

(b) $\int \cos 4x \;\; \ln(\sin 4x)\; dx = \dfrac{1}{4} \int \ln(\sin 4x)\;\; \cos 4x \; 4\, dx = ?$ by #27

$$\frac{1}{4}[\sin 4x \;\; \ln(\sin 4x) \;-\; \sin 4x] \;+\; C$$

(c) $\int \sec^2 x \;\; \ln(\tan x)\; dx$

(d) $\int x \;\; \ln(x^2+4)\; dx$

$$\frac{1}{2}[(x^2+4)\;\; \ln(x^2+4) \;-\; (x^2+4)] + C$$

29. Integrate by parts to find each integral.

(a) $\int x^3 e^{x^2}\, dx = \int (x^2)\, [e^{x^2}\; x\, dx]$

(b) $\int x^3 \cos x^2\; dx = \int (x^2)\, [\cos x^2\; x\, dx]$

(c) $\int x^3 \sqrt{4+x^2}\; dx = \int (x^2)\, [(4+x^2)^{\frac{1}{2}}\; x\, dx]$

(d) $\int \dfrac{x^3\; dx}{\sqrt[3]{4+x^2}} = \int (x^2)\, [(4+x^2)^{-\frac{1}{3}}\; x\, dx]$

30. Find each integral.

(a) $\int e^{\sqrt{x}}\; dx$

(b) $\int \cos\sqrt{x}\; dx$

If it is frustrating to integrate these two by parts as they are, try a substitution. Substitute $t = \sqrt{x}$. Then $x = t^2$ and so $dx = 2t\, dt$. Then integrate by parts.

31. Do Exercise 50 at the end of Section 7.1 in the textbook.

32. Show each formula. We will do one in class; you do the other similarly.

$$\int e^{ax}\, \cos bx\; dx = \frac{e^{ax}}{a^2+b^2}\,(a\, \cos bx + b\, \sin bx) + C$$

$$\int e^{ax}\, \sin bx\; dx = \frac{e^{ax}}{a^2+b^2}\,(a\, \sin bx - b\, \cos bx) + C$$

On our first test, we will be asked to do #31 or one part of #32.

33. $\int x^3\, e^x\; dx$ Integrate by parts three times.

$$(x^3 - 3x^2 + 6x - 6)\, e^x + C$$

34. $\int x^2\, \cos 2x\; dx$ parts 2 times $\qquad \dfrac{x^2}{2}\, \sin 2x + \dfrac{x}{2}\, \cos 2x - \dfrac{1}{4}\, \sin 2x + C$

In the next problem, use long division to divide x^2 by x^2+1 and get $1 - \dfrac{1}{x^2+1}$.

35. $\int x\, \arctan x\; dx = \int (\arctan x)\, [x\; dx]$

$$= \frac{x^2}{2}\, \arctan x \; - \int \frac{x^2}{2}\, \frac{dx}{x^2+1} = \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right)\, dx = ?$$

36. $\int (\arcsin x)\ [dx]$ $\qquad x \arcsin x \ + \ \sqrt{1-x^2} + C$

37. $\int (\arctan x)\ [dx]$ $\qquad x \arctan x \ - \ \frac{1}{2}\ln(1+x^2) \ + \ C$

38. $\int \sin^8 x \cos^3 x\ dx = \int \sin^8 x\ \ \cos^2 x\ \ \cos x\ dx$

$$= \int \sin^8 x\,(1-\sin^2 x)\ \cos x\ dx = \int u^8(1-u^2)du = ?$$

39. $\int (\sin x)^{-\frac{1}{2}} \cos^5 x\ dx = \int u^{-\frac{1}{2}}(\cos^2 x)^2 \cos x\ dx$

$$= \int u^{-\frac{1}{2}}(1-u^2)^2 du = ?$$

Study Red Box 1 in Sec 7.2.

40. $\int \sin^2 x\ dx = \int \frac{1}{2}\,(1-\cos 2x)\ dx = ?$

If the quadratic expression ax^2+bx+C with a negative or fractional exponent on this whole expression appears in an integrand, we often complete the square and make a little substitution.

41.

$$\begin{aligned}
\int \frac{(3x+7)dx}{\sqrt{7+6x-x^2}} &= \cdots = \int \frac{[3(t+3)+7]dt}{\sqrt{16-(t)^2}} \\
&= 3\int \frac{t\,dt}{(16-t^2)^{\frac{1}{2}}} + 16\int \frac{dt}{\sqrt{4^2-t^2}} \\
&= \cdots = \left(-\frac{3}{2}\right)\left(\frac{2}{1}\right)(16-t^2)^{\frac{1}{2}} \\
&\quad + 16\arcsin\frac{t}{4} + C \\
&= -3\sqrt{16-(x-3)^2} + 16\arcsin\frac{x-3}{4} + C \\
&= -3\sqrt{7+6x-x^2} + 16\arcsin\frac{x-3}{4} + C
\end{aligned}$$

42.

$$\begin{aligned}
\int \frac{3x+5}{x^2-8x+20}\,dx &= \int \frac{3x+5}{(x-4)^2+4}\,dx \\
&= \int \frac{3(t+4)+5}{(t)^2+4}\,dt \\
&= 3\int \frac{t\,dt}{(t)^2+4} + 17\int \frac{dt}{2^2+t^2} \ = \cdots
\end{aligned}$$