Section:

MAC 2313 Quiz 1

Name: Key

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Show all work, no calculators or cellphones.

1. (2 points) Find the distance $d(A, B)$ between $A(3, -2, -3)$ and $B(7, 0, 1)$. Find the magnitude of $\vec{AB}$.

   $d(A,B) = \sqrt{(7-3)^2 + (0--2)^2 + (1--3)^2} = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{16+4+16} = \sqrt{36} = \boxed{6}$

   $|\vec{AB}| = \boxed{6}$

2. (2 points) If $\vec{u} = \langle 1, 2, 3\rangle$ and $\vec{v} = 2\vec{i} + 7\vec{j} + 5\vec{k}$ find $\vec{u} + \vec{v}$.

   $\langle 2, 7, 5 \rangle$

   $\vec{u} + \vec{v} = \langle 1+2, 2+7, 3+5 \rangle = \langle 3, 9, 8 \rangle$

3. Determine if the following vectors are orthogonal. If not find the cosine of the angle between them.

   (a) (2 points) $\vec{s} = \langle -5, 3, 7\rangle$ and $\vec{t} = \langle 6, -8, 2\rangle$.

   $\vec{s} \cdot \vec{t} = |s||t| \cos\theta$

   $\vec{s} \cdot \vec{t} = -5 \cdot 6 + 3 \cdot -8 + 7 \cdot 2 = -30 - 24 + 14 = -40$

   $|\vec{s}| = \sqrt{(-5)^2 + 3^2 + 7^2} = \sqrt{25 + 9 + 49} = \sqrt{83}$

   $|\vec{t}| = \sqrt{6^2 + (-8)^2 + 2^2} = \sqrt{36 + 64 + 4} = \sqrt{104}$

   $\cos\theta = \frac{\vec{s} \cdot \vec{t}}{|s||t|} = \boxed{\frac{-40}{\sqrt{83}\sqrt{104}}}$

   (not orthogonal)

   (b) (2 points) $\vec{x} = -\vec{i} + 2\vec{j} + 5\vec{k}$ and $\vec{y} = 3\vec{i} + 4\vec{j} - \vec{k}$

   $= \langle -1, 2, 5 \rangle$ $= \langle 3, 4, -1 \rangle$

   $\vec{x} \cdot \vec{y} = -1 \cdot 3 + 2 \cdot 4 + 5 \cdot -1 = -3 + 8 - 5 = 0$

   $\vec{x} \cdot \vec{y} = 0$ so $\vec{x}$ and $\vec{y}$ $\boxed{\text{are orthogonal}}$

4. (2 points) If $\vec{a} = \langle -2, 3, -6\rangle$ and $\vec{b} = \langle 1, 2, 3\rangle$ find the projection of $\vec{b}$ onto $\vec{a}$.

   $\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|a|^2} \vec{a} = \frac{-2 \cdot 1 + 3 \cdot 2 + -6 \cdot 3}{(-2)^2 + 3^2 + (-6)^2} \vec{a} = \frac{-2 + 6 - 18}{4 + 9 + 36} \vec{a} = \frac{-14}{49} \vec{a} = \boxed{-\frac{2}{7} \vec{a}}$

   $\boxed{= -\frac{2}{7} \langle -2, 3, -6 \rangle = \langle \frac{+4}{7}, -\frac{6}{7}, \frac{12}{7} \rangle}$

BONUS! Show that $\text{orth}_{\vec{a}} \vec{b} = \vec{b} - \text{proj}_{\vec{a}} \vec{b}$ is orthogonal to $\vec{a}$

we need $\text{orth}_{\vec{a}} \vec{b} \cdot \vec{a}$