

(Rational Function)

Remember to use $\ln|u|$ where appropriate.

1. $\int_0^1 \frac{x^3 - x - 8}{x^2 - x - 6} dx \quad \left(\frac{3}{2} - \frac{2}{5} \ln\left(\frac{3}{2}\right)\right).$

2. $\int \frac{2x^2 - 11x + 19}{(2x + 1)(x - 2)^2} dx \quad \left(2 \ln(|2x + 1|) - \ln(|x - 2|) - \frac{1}{x - 2} + C\right).$

3. $\int \frac{x^3 + 1}{x^2 + 9} dx \quad \left(\frac{1}{2}x^2 - \frac{9}{2} \ln(x^2 + 9) + \frac{1}{3} \arctan\left(\frac{1}{3}x\right) + C\right).$

4. $\int \frac{34}{(x - 5)(x^2 + 9)} dx \quad \left(\ln(|x - 5|) - \frac{1}{2} \ln(x^2 + 9) - \frac{5}{3} \arctan\left(\frac{x}{3}\right) + C\right).$

5. $\int \frac{x^2 + 5x + 1}{(x^2 + 1)^2} dx \quad \left(\arctan x - \frac{5}{2x^2 + 2} + C\right).$

6. $\int \frac{7}{x^3 - 1} dx \quad \left(\frac{7}{3} \ln(|x - 1|) - \frac{7}{6} \ln(x^2 + x + 1) - \frac{7}{\sqrt{3}} \arctan\left(\frac{1}{\sqrt{3}}(2x + 1)\right) + C\right).$

7. $\int_1^3 \frac{x}{x^2 + 4x + 13} dx \quad \left(\frac{1}{2} \ln\left(\frac{17}{9}\right) - \frac{2}{3} \left(\arctan\left(\frac{5}{3}\right) - \frac{\pi}{4}\right)\right).$

8. $\int \frac{3x^3 + 18x^2 + 33x - 2}{(x^2 + 6x + 10)^2} dx \quad \left(\frac{3}{2} \ln(x^2 + 6x + 10) - \frac{29}{2} \arctan(x + 3) - \frac{11x + 36}{2x^2 + 12x + 20} + C\right).$

9. $\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}} dx \quad \left(2x^{1/2} + 3x^{1/3} + 6x^{1/6} + 6 \ln(|x^{1/6} - 1|) + C\right).$

(hint, make a substitution first to express the integrand as a rational function and then evaluate the integral. let $u = \sqrt[6]{x}$)

10. $\int \frac{\cos(x) dx}{7 \sin^2(x) + 2 \sin(x)} \quad \left(\frac{1}{2} \ln\left|\frac{\sin x}{7 \sin x + 2}\right|\right) + C).$
(hint, make a substitution first)

11. $\int \frac{e^x}{(e^x - 9)(e^{2x} + 16)} dx \quad \left(\frac{1}{97} \ln(|e^x - 9|) - \frac{1}{194} \ln(|e^{2x} + 16|) - \frac{9}{388} \arctan\left(\frac{e^x}{4}\right) + C\right).$
(hint, make a substitution first)

12. $\int 11 \ln(x^2 - x + 6) dx \quad \left(11\left(x - \frac{1}{2}\right) \ln(x^2 - x + 6) - 22x + 11(23^{1/2}) \arctan\left(\frac{1}{23}(2x - 1)23^{1/2}\right) + C\right).$
(hint, use integration by parts, with the technics of this section)

13. $\int \frac{dx}{x^2 - 4} \quad \left(\frac{\ln\left|\frac{x-2}{x+2}\right|\right)}{4} + C).$

14. $\int \frac{x+1}{x^2+5x+4} dx \quad (\ln(|x+4|) + C).$

15. $\int \frac{x^2+x+7}{(x^2+7)^2} dx \quad \left(\frac{\arctan\left(\frac{x}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{1}{2(x^2+7)} + C\right).$

16. $\int \frac{8x^3}{x^2+1} dx \quad (4x^2 = 4 \ln(x^2+1) + C).$

17. $\int_1^3 \frac{6x-1}{x^2(x+1)} dx \quad (7 \ln(3/2) - 2/3).$

18. $\int_7^8 \frac{x dx}{x^2-4x-5} \quad \left(\frac{1}{6} \ln\left(\frac{2187}{256}\right) + C\right).$

19. $\int \frac{6x-4}{3x^2-4x+7} dx$