Computer Project 4: Optimization

Due: December 5, 2007

In this project, you have to write a MATLAB code for finding a minimal road connecting four points A, B, C, D in the plane. The theory (which I will explain in more detail in class) states that such minimal net can be obtained by adding at most two points $X_1 = (x_1, y_1)$ and $X_2 = (x_2, y_2)$ so that the net itself consists of at most five segments: $[A, X1]$, $[B, X1]$, $[C, X2]$, $[D, X_2]$, and $[X1, X2]$, where the original points may have to renamed appropriately. In some cases, the actual optimum may be achieved by adding just one point X.

What I want you to do is to minimize two objective functions:

- $F_1(x, y)$ which is the sum of four distances from X to A, B, C, D;
- $F_2(x_1, y_1, x_2, y_2)$ which is the sum of five distances $[A, X1]$, $[B, X1]$, $[C, X2]$, $[D, X_2]$, and $[X1, X2]$; this function must be minimized over all possible pairings of points A, B, C, D of which there are six.

Once both minima are found, you should choose the smaller of the two, and in doing so decide which type of the road gives minimal length.

Your script should work as a function called `road(coords)` where `coords` is a vector of 8 coordinates of the points A, B, C, D. You are allowed to use any built-in MATLAB minimizers.

The code should display the graph of the minimal net showing the positions of the vertices and the connecting segments. The code should also output the length of the minimal road.