Review for Final Exam

Mathematical Methods for Life Sciences, Fall 2007

The final exam will be held on Monday, December 10 at 7:30 am in LIT 221 (you may also take it during alternate time: Thursday, December 14 at 12:30 pm in LIT 219). The final exam will be cumulative and it will include 4-5 computational problems as well as 1-2 theoretical questions (definitions, statements, descriptions, etc.). The final exam will not test MATLAB skills. Use of calculators will be allowed.

For the **theoretical part**, you should revisit the following concepts and notions (that we have discussed in class): vectors and matrices, vectors norms and distance functions, complex numbers, mean and variance, correlation coefficient, discrete/continuous random variables, discrete probability function, cumulative distribution function, probability density function, conditional probability, Bayes formula, independence of random variables, random walk, Poisson process,simple birth-and-death process, Kolmogorov-Chapman equations, matrix operations, linear system of equations, RREF of a matrix, Gaussian elimination, matrix inverses, span, linear independence, basis, column space, null space, linear transformation, projection, matrix determinant, eigenvalues and eigenvectors, discrete and continuous dynamical system, geometric growth/decay, fixed points and equilibria, objective function, feasible set, local and global minima/maxima, critical points, gradient of a multivariate function.

For the **computational part**, you will need to know how to solve problems of the following types:

1. A light bulb typically lasts for 6 months before it has to be replaced. What is the probability that you'll have to replace the bulb after only 2 months of use? (exponential distribution)

2. A box contains 12 shirts of which 3 have blemishes and the rest are good. What is the probability that if two shirts are selected at random from the box, both will have blemishes?

3. A hotel chain gets cars for its guests from three rental agencies: 25% from agency X, 40% from agency Y, and the rest from agency Z. If 15% of all cars from agency X need tune-ups, 10% from agency Y need tune-ups, and 5% from agency Z need tune-ups, what is the probability that a car that needs a tune-up when its delivered to a guest came from agency X?

4. Solve the linear system with the augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 1 \\ 1 & 0 & -2 & 1 & 2 \\ 0 & 1 & 2 & -1 & 3 \end{array}\right]$$

5. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

6. Find the matrix P of the orthogonal projection onto the plane $x + y + z = 0$.
7. Find the global maximum and minimum of the function $f(x) = x(1-x)(0.5-x)$ on the interval $[0, 1]$.
8. A farmer is allotted a certain area of land A. How should he choose a rectangular lot so that the amount of fence required to enclose the lot would be minimized.
9. Find the dimensions of a rectangular box of maximal volume such that the sum of the lengths of its 12 edges is a constant $12L$.
10. Find the dimensions of the rectangle of the largest area that fits into a circle of radius R.

Key.

1. $P(x \le 2) = 1 - e^{-1/3} \approx 0.283$;
2. $P(2) = \binom{3}{2} / \binom{12}{2} = 1/22 \approx 0.045$;
3. $P(X|\text{tuneup}) = \frac{0.15\times0.25}{0.15\times0.25+0.1\times0.4+0.05\times0.35} \approx 0.395$;
4. $x_1 = 16 + 3t,\ x_2 = -11 - 3t,\ x_3 = 7 + 2t,\ x_4 = t$, t arbitrary;
5. $\lambda_1 = -1$, $\mathbf{v}_1 = (1, -1)$; $\lambda_2 = 3$, $\mathbf{v}_2 = (1, 1)$;
6. $$P = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}$$
7. $f_{\max} = \sqrt{3}/36$ is attained at $x = 1/2 - \sqrt{3}/6$;
   $f_{\min} = -\sqrt{3}/36$ is attained at $x = 1/2 + \sqrt{3}/6$;
8. $x = y = \sqrt{A}$, $L_{\min} = 4\sqrt{A}$;
9. $x = y = z = L$, $V_{\max} = L^3$;
10. $x = y = \sqrt{2}R$, $A_{\max} = 2R^2$;