

SHOW ALL WORK TO RECEIVE FULL CREDIT.

- (2.5 points) Decompose to partial fractions:  $f(x) = \frac{1 - 6x}{x^2(x + 1)}$ .
- (2.5 points) Find  $\int \frac{x - 4}{x^2 + 9} dx$ .

Solution:

1. 
$$\frac{1 - 6x}{x^2(x + 1)} = \frac{A}{x + 1} + \frac{B}{x} + \frac{C}{x^2}$$
$$1 - 6x = Ax^2 + Bx(x + 1) + C(x + 1).$$

Let  $x = -1$ :  $7 = A$ .

Let  $x = 0$ :  $1 = C$ .

Let  $x = 1$ :  $-5 = A + 2B + 2C$ , thus  $B = -7$ .

So 
$$\frac{1 - 6x}{x^2(x + 1)} = \frac{7}{x + 1} - \frac{7}{x} + \frac{1}{x^2}.$$

2. 
$$\int \frac{x - 4}{x^2 + 9} dx = \int \frac{x}{x^2 + 9} dx - \int \frac{4}{x^2 + 9} dx$$
$$\stackrel{(u=x^2+9)}{=} \int \frac{\frac{1}{2} du}{u} - 4 \int \frac{dx}{x^2 + 3^2}$$
$$= \frac{1}{2} \ln |u| - 4 \cdot \frac{1}{3} \arctan \frac{x}{3} + C$$
$$= \frac{1}{2} \ln(x^2 + 9) - \frac{4}{3} \arctan \frac{x}{3} + C.$$