MAC2233 Final A

(7 pts) 1. Given $y^2x^2 + 3x = 3y^3$, y' is given by:

A. $\frac{2xy^2+3}{9y^2-2yx^2}$ B. $\frac{2xy^2+3}{9y^2+2yx^2}$ C. $\frac{2xy^2-3}{9y^2-2yx^2}$ D. $\frac{2xy^2+3}{9y^3+2yx^2}$ E. $\frac{2xy^2+3}{9y^2-2y^2x^2}$

(7 pts) 2. Given $h(x) = (4x^2 - 3x\ln x)^3$, $h'(1)$ is equal to:

A. 120 B. 80 C. 240 D. 180 E. 360

(7 pts) 3. The definite integral $\int_{-2}^{2}\ 5x^3 - 6x\ dx$ is equal to:

A. $\frac{11}{6}$ B. -$\frac{11}{6}$ C. $\frac{5}{6}$ D. $\frac{7}{6}$ E. 0

(7 pts) 4. An investor places \$3000 in an account which compounds quarterly at a standard rate of 8%; how long will it take for his investment to double?

A. $\frac{\ln 2}{4\ln 1.02}$ years B. $\frac{\ln 6000}{3000\ln 1.02}$ years C. $\frac{\ln 6000}{12{,}000\ln 1.02}$ years

D. $\frac{\ln 2}{4\ln 3}$ years E. $\frac{\ln 2}{\ln 1.02}$ years

(7 pts) 5. Given $f(x) = 2xe^{-x}$, which of the following statements are true:

I. $f(x) \geq 0$ for all x

II. the function is decreasing for $x > 1$

III. the function is concave upward on $(2, +\infty)$

IV. there is an inflection point at $x = 2$

A. only III B. only II and III C. only II and IV D. only II, III, and IV

E. I, II, III, and IV

(7 pts) 6. The area of the region completely enclosed by the graphs of $f(x) = x^2 - 3x$ and $g(x) = -x$ is given by :

A. $\frac{1}{3}$ B. $\frac{4}{3}$ C. $\frac{5}{3}$ D. $\frac{2}{3}$ E. $\frac{7}{3}$

(7 pts) 7. The maximum value of the function $f(x) = x^3 - 2x^2 - 4x + 4$ on the interval $[0, 3]$ is given by:

A. 1 B. -4 C. 4 D. 6 E. -8

(7 pts) 8. Approximate the area under the graph of $g(x) = -4x + 8$ over the interval $[-1, 2]$ by using a Riemann sum, subdividing $[-1, 2]$ into six subintervals of equal length, and choosing the right endpoint of each subinterval as the sample point. The approximation is equal to:

A. 20 B. 30 C. 15 D. 24 E. 42

(7 pts) 9. A coffee pot in the form of a circular cylinder of radius 4 in is being filled with water at a constant rate. If the water level is rising at a rate of 0.4 in/sec, how fast is water flowing into the pot? (Note the volume of a cylinder is given by $V = \pi r^2 h$):

A. $5.6\pi\ in^3/sec$ B. $6.4\pi\ in^3/sec$ C. $12.8\pi\ in^3/sec$ D. $4.8\pi\ in^3/sec$ E. $2.4\pi\ in^3/sec$

(7 pts) 10. The average rate of change of the function $f(x) = 4x \ln x$ on the interval $[1,\ e]$ is given by:

A. 4 B. $\frac{4}{e-1}$ C. $\frac{4e}{e-1}$ D. $\frac{4(e+1)}{e-1}$ E. $\frac{4e(e+1)}{e-1}$

(7 pts) 11. The function $g(x)$ has vertical asymptotes at $x = \pm 1$ and a horizontal asymptote at $y = 2$; is concave upward on $(-\infty, -1) \cup (1, +\infty)$ and concave downward on $(-1, 1)$; and is increasing on $(-\infty, -1) \cup (-1, 0)$ and decreasing on $(0, 1) \cup (1, \infty)$. Which of the following is equal to $g(x)$?

A. $\frac{2x^2}{(x-1)^2}$ B. $\frac{2(x^2+3x+2)}{(x-1)(x+1)}$ C. $\frac{2x}{(x-1)(x+1)}$ D. $\frac{2x^2}{x^3-1}$ E. $\frac{2x^2}{x^2-1}$

(7 pts) 12. The equation of the tangent line to the function $f(x) = \sqrt{x}(2x+4)$ at $x = 4$ is given by

A. $y = 5x + 4$ B. $y = 7x - 4$ C. $y = 4x + 8$ D. $y = 3x + 12$ E. $y = 6x$

(7 pts) 13. Let x and y be any two positive numbers such that $xy = 80$; the smallest possible value of the sum $4x + 20y$ is given by:

A. 80 B. 120 C. 160 D. 180 E. 200

(7 pts) 14. Which of the following applications of the chain rule is (are) correct:

I. $\frac{d}{dx}\ \ln(f(x)) = \frac{f'(x)}{f(x)}$

II. $\frac{d}{dx}\ e^{f(x)} = f'(x)\ e^{f(x)}$

III. $\frac{d}{dx}\ [f(x)]^n = n[f(x)]^{n-1} f'(x)$

A. only III B. only II and III C. only I and III D. only I and II E. I, II, and III

(7 pts) 15. A calculator manufacturer has an estimated marginal profit given by

$$P'(x) = -0.5x + 100$$

where the units for $P'(x)$ are dollars/calculator/month when the production level is x calculators per month. If the fixed cost for producing and selling these calculators is \$6,000/month, the maximum monthly profit is:

A. \$5,000 B. \$4,000 C. \$6,000 D. \$4,500 E. \$5,500

Solutions:

1. A 2. C 3. E 4. A 5. D 6. B 7. C 8. C 9. B 10. C 11. E 12. B

13. C 14. E 15. B