MAC2312 Test 1

1. The area bounded by the curves $y = x(x-2)$ and $y = x$ is given by the integral:

A. $\int_0^1 \ x(x-2) - x \ dx$ B. $\int_0^3 \ x(x-2) - x \ dx$ C. $\int_0^3 \ 3x - x^2 \ dx$

D. $\int_0^9 \ 3x - x^2 \ dx$ E. $\int_0^3 \ 3x + x^2 \ dx$

2. The volume enclosed by revolving the curve $y = e^{-x}, \ x \in [0, \pi]$, about the x axis is given by the integral:

A. $\int_0^\pi \ \pi e^{-x} \ dx$ B. $\int_0^\pi \ \pi e^{-2x} \ dx$ C. $\int_0^\pi \ \pi^2 e^{-x} \ dx$ D. $\int_0^\pi \ (\pi e^{-x})^2 \ dx$ E. $\int_0^{ln\pi} \ \pi(\ln y)^2 \ dy$

3. The indefinite integral $\int x^2 \ln x \ dx$ is equal to:

A. $\frac{1}{3}x^3 \ln x - \frac{1}{6}x^3 + C$ B. $\frac{1}{6}x^3 \ln x - \frac{1}{9}x^3 + C$ C. $\frac{1}{3}x^3 (\ln x)^2 - \frac{1}{6}x^3 + C$

D. $\frac{1}{3}x^3 \ln x + \frac{1}{6}x^3 + C$ E. $\frac{1}{3}x^3(\ln x - \frac{1}{3}) + C$

4. The definite integral $\int_0^\pi \sin^2 x \cos^2 x \; dx$ is equal to:

A. $\pi/8$ B. 0 C. $\pi/4 - 1/4$ D. $\pi/8 + 1/16$ E. none of the above

5. Given a spring with a natural length of 10 cm and a spring constant of 3 N/m, how much work is done in stretching the spring from 12 to 14 cm?

A. $\int_{12}^{14} 3x \; dx$ B. $\int_2^4 3x \; dx$ C. $\int_2^4 0.3x \; dx$ D. $\int_{0.2}^{0.4} 3x^2 \; dx$ E. $\int_{0.02}^{0.04} 3x \; dx$

6. The average value of the function $y = e^{3x}$ over the interval $[0, 2]$ is given by:

A. $\frac{1}{2}(e^3 - e^2)$ B. $\frac{1}{6}e^6$ C. $\frac{1}{2}e^6$ D. $\frac{1}{6}(e^6 - 1)$ E. $\frac{1}{3}(e^6 - 1)$

7. Use the method of washers to find the volume obtained by revolving the region bounded by the curve $y = x^2$, the line $x = 1$, and the x axis about the y axis.

8. A 4 kg bucket is attached to one end of a 6 m chain which has a mass of 2 kg/m; if the other end of the chain is attached to a 10 m platform, determine the work done in lifting the chain and bucket to the top of the platform.

Solutions:

1. C   2. B   3. E   4. A   5. E   6. D

7. For this problem, the height of the washer is given by $h = dy$, the inner radius is given by $r_i = x = \sqrt{y}$, and the outer radius is given by $r_o = 1$. Therefore the volume is

$$V = \int_0^1 \pi(1^2 - (\sqrt{y})^2)\ dy$$
$$V = \pi \int_0^1 1 - y\ dy$$
$$V = \pi(y - y^2/2)|_0^1$$
$$V = \pi[(1 - 1^2/2) - (0 - 0^2/2)]$$
$$V = \pi/2.$$

8. For this problem we will let y denote the distance which the end of the chain attached to the bucket has been lifted. Initially $y = 0$ and after the chain and bucket have been lifted, $y = 6$. We will determine the differntial work dW done in lifting the remaining chain and bucket a distance of dy meters for some $0 < y < 6$. The length of the chain remaining is $(6 - y)\ m$ and this has a mass of $2(6 - y)\ kg$; hence the total mass of the system at this point is $[2(6 - y) + 4]\ kg$ and the graviattional force acting on it is $[2(6 - y) + 4]g\ N$ where $g = 9.8\ m/s^2$. Hence the work dW required to lift the system dy meters is given by

$$dW = [2(6 - y) + 4]gdy\ J$$

and the total work required is

$$W = \int\ dW = \int_0^6 [2(6 - y) + 4]g\ dy$$
$$W = g\int_0^6 16 - 2y\ dy$$
$$W = g(16y - y^2)|_0^6$$
$$W = g[(16(6) - 6^2) - (16(0) - 0^2)]$$
$$W = g(60) = 9.8(60)\ J.$$