MAC2311 Lab 3: Basic Derivatives

Find the first derivatives of the given functions.

1. $y = \sqrt[3]{x}$

2. $y = \sqrt[2]{x}(x - 1)$

3. $f(t) = ae^t + \frac{b}{t} + \frac{c}{t^2}$

4. $R(z) = \frac{e^z}{1+e^z}$

5. $h(x) = (x - 2)(2x + 3)$

6. $y = \frac{3x^2-4x+2}{x^2+2}$

7. $h(r) = (3r^2 - 2)^2$

8. $y = \frac{x^2-3e^x}{8}$

9. $y = x^2 e^x$

10. $y = \frac{\sqrt{x}e^x}{2+xe^x}$

11. Find the first, second, and third derivatives of $f(x) = -3x^2 + 7x - 2$.

12. Find the first and second derivatives of $f(x) = \frac{2e^x}{3x+5}$.

13. Find the first, second, and third derivatives of $f(x) = 3x^2 e^x$.

Solutions:

1. $y' = \frac{d}{dx}\ \sqrt[3]{x} = \frac{d}{dx}\ x^{(1/3)} = x^{-(2/3)}/3.$

2. $y' = \frac{d}{dx}\ \sqrt[2]{x}(x-1) = \frac{d}{dx}\ x^{(1/2)}(x-1) = \frac{d}{dx}\ (x^{(3/2)} - x^{(1/2)}) = 3x^{(1/2)}/2 - x^{-(1/2)}/2$

$= 3\sqrt{x}/2 - 1/(2\sqrt{x}).$

3. $f'(t) = \frac{d}{dt}\ (ae^t + \frac{b}{t} + \frac{c}{t^2}) = \frac{d}{dt}\ (ae^t + bt^{-1} + ct^{-2}) = ae^t - bt^{-2} - 2ct^{-3}.$

4. $R'(z) = \frac{d}{dz}\ \frac{e^z}{1+e^z} = \frac{(e^z)'\cdot(1+e^z)-(e^z)\cdot(1+e^z)'}{(1+e^z)^2} = \frac{(e^z)\cdot(1+e^z)-(e^z)\cdot(e^z)}{(1+e^z)^2} = \frac{e^z+e^{2z}-e^{2z}}{(1+e^z)^2} = \frac{e^z}{(1+e^z)^2}.$

5. $h'(x) = \frac{d}{dx}\ (x-2)(2x+3) = \frac{d}{dx}\ (2x^2+3x-4x-6) = \frac{d}{dx}\ (2x^2-x-6) = 4x-1.$

Or using the product rule:
$h'(x) = \frac{d}{dx}\ (x-2)(2x+3) = (x-2)'\cdot(2x+3)+(x-2)\cdot(2x+3)' = (1)(2x+3)+(x-2)(2) = 4x-1.$

6. $y' = \frac{d}{dx}\ \frac{3x^2-4x+2}{x^2+2} = \frac{(3x^2-4x+2)'\cdot(x^2+2)-(3x^2-4x+2)\cdot(x^2+2)'}{(x^2+2)^2} = \frac{(6x-4)\cdot(x^2+2)-(3x^2-4x+2)\cdot(2x)}{(x^2+2)^2}$

$= \frac{(6x^3-4x^2+12x-8)-(6x^3-8x^2+4x)}{(x^2+2)^2} = \frac{4x^2+8x-8}{(x^2+2)^2} = \frac{4(x^2+2x-2)}{(x^2+2)^2}.$

7. $h'(r) = \frac{d}{dr}\ (3r^2-2)^2 = \frac{d}{dr}\ (9r^4-12r^2+4) = 36r^3-24r.$

Or using the product rule:
$h'(r) = \frac{d}{dr}\ (3r^2-2)^2 = (3r^2-2)'\cdot(3r^2-2)+(3r^2-2)\cdot(3r^2-2)' = (6r)(3r^2-2)+(3r^2-2)(6r)$

$= (12r)(3r^2-2) = 36r^3-24r.$

8. $y' = \frac{d}{dx}\ \frac{x^2-3e^x}{8} = \frac{d}{dx}\ (x^2/8 - 3e^x/8) = x/4 - 3e^x/8.$

9. $y' = \frac{d}{dx}\ x^2e^x = (x^2)'(e^x)+(x^2)(e^x)' = (2x)(e^x)+(x^2)(e^x) = (x^2+2x)e^x.$

10. $y' = \frac{d}{dx}\ \frac{\sqrt{x}e^x}{2+xe^x} = \frac{(\sqrt{x}e^x)'\cdot(2+xe^x)-(\sqrt{x}e^x)\cdot(2+xe^x)'}{(2+xe^x)^2} = \frac{(x^{(1/2)}e^x)'\cdot(2+xe^x)-(x^{(1/2)}e^x)\cdot(2+xe^x)'}{(2+xe^x)^2}$

$= \frac{[(x^{(1/2)})'(e^x)+(x^{(1/2)})(e^x)']\cdot(2+xe^x)-(x^{(1/2)}e^x)\cdot[(x)'(e^x)+(x)(e^x)']}{(2+xe^x)^2}$

$$= \frac{[(x^{(-1/2)}/2)(e^x)+(x^{(1/2)})(e^x)]\cdot(2+xe^x)-(x^{(1/2)}e^x)\cdot[(1)(e^x)+(x)(e^x)]}{(2+xe^x)^2}$$

$$= \frac{x^{(-1/2)}e^x+x^{(1/2)}e^{2x}/2+2x^{(1/2)}e^x+x^{(3/2)}e^{2x}-x^{(1/2)}e^{2x}-x^{(3/2)}e^{2x}}{(2+xe^x)^2}$$

$$= \frac{x^{(-1/2)}e^x-x^{(1/2)}e^{2x}/2+2x^{(1/2)}e^x}{(2+xe^x)^2} = \frac{(x^{(1/2)}e^x)(x^{-1}-e^x/2+2)}{(2+xe^x)^2} = \frac{(x^{(1/2)}e^x)(2-xe^x+4x)}{(2x)(2+xe^x)^2} = \frac{e^x(2-xe^x+4x)}{2\sqrt{x}(2+xe^x)^2}.$$

11. $f'(x) = \frac{d}{dx}\ (-3x^2+7x-2) = -6x+7.$

$f''(x) = \frac{d}{dx}\ f'(x) = \frac{d}{dx}(-6x+7) = -6.$

$f'''(x) = \frac{d}{dx}\ f''(x) = \frac{d}{dx}(-6) = 0.$

12. $f'(x) = \frac{d}{dx}\ \frac{2e^x}{3x+5} = \frac{(2e^x)'(3x+5)-(2e^x)(3x+5)'}{(3x+5)^2} = \frac{(2e^x)(3x+5)-(2e^x)(3)}{(3x+5)^2} = \frac{6xe^x+10e^x-6e^x}{(3x+5)^2}$

$= \frac{6xe^x+4e^x}{(3x+5)^2}.$

$$f''(x) = \frac{d}{dx}\ f'(x) = \frac{d}{dx}\ \frac{6xe^x+4e^x}{(3x+5)^2} = \frac{(6xe^x+4e^x)'((3x+5)^2)-(6xe^x+4e^x)((3x+5)^2)'}{(3x+5)^4}$$

$$= \frac{(6xe^x+6e^x+4e^x)((3x+5)^2)-(6xe^x+4e^x)(9x^2+30x+25)'}{(3x+5)^4} = \frac{(6xe^x+10e^x)((3x+5)^2)-(6xe^x+4e^x)(18x+30)}{(3x+5)^4}$$

$$= \frac{(6xe^x+10e^x)((3x+5)^2)-(6xe^x+4e^x)(6)(3x+5)}{(3x+5)^4} = \frac{(3x+5)[(6xe^x+10e^x)(3x+5)-6(6xe^x+4e^x)]}{(3x+5)^4}$$

$$= \frac{[18x^2e^x+30xe^x+30xe^x+50e^x-36xe^x-24e^x]}{(3x+5)^3} = \frac{[18x^2e^x+24xe^x+26e^x]}{(3x+5)^3} = \frac{2e^x(9x^2+12x+13)}{(3x+5)^3}.$$

13. $f'(x) = \frac{d}{dx}\ 3x^2e^x = (3x^2)'(e^x)+(3x^2)(e^x)' = (6x)(e^x)+(3x^2)(e^x) = (6x+3x^2)e^x.$

$f''(x) = \frac{d}{dx}\ f'(x) = \frac{d}{dx}\ (6x+3x^2)e^x = (6x+3x^2)'(e^x)+(6x+3x^2)(e^x)'$

$= (6+6x)(e^x)+(6x+3x^2)(e^x) = (3x^2+12x+6)e^x.$

$f'''(x) = \frac{d}{dx}\ f''(x) = \frac{d}{dx}\ (3x^2+12x+6)e^x = (3x^2+12x+6)'(e^x)+(3x^2+12x+6)(e^x)'$

$= (6x+12)(e^x)+(3x^2+12x+6)(e^x) = (3x^2+18x+18)e^x.$