MAC2311 Lab 9: Antiderivatives and Riemann Sums

Find the antiderivatives of the following functions:

1. $f(x) = 7x^3$

2. $g(x) = 3\cos x - 2\sin x$

3. $r(t) = -5e^t + 4\sec t \tan t$

4. $f(x) = 6\sqrt{x} - 9/x^2$

5. $h(t) = 4\cos^2(\frac{t}{2})$

6. $g(x) = \frac{2x^2-7}{x^3}$

7. $h(x) = (x+2)^3$

8. $s(x) = \frac{-5}{x^2+1}$

9. Given $f''(x) = 6x^2 + 12x + 5$, find $f(x)$ such that $f(0) = 1,\ f'(0) = -1$.

10. Given $f''(x) = \sin x - \cos x$, find $f(x)$ such that $f(\pi) = \pi,\ f'(\pi) = 1$.

11. Approximate the area under the curve $y = 3x,\ x \in [0, 8]$ using four rectangles of equal width and the midpoint of each resulting interval as the sample point. What is the actual area?

12. Approximate the area under the curve $y = x^2 + 1,\ x \in [-5, 15]$ using five rectangles of equal width and the left endpoint of each resulting interval as the sample point. Repeat your calculations using the right endpoint of each interval.

Solutions:

1. The antiderivative is given by $F(x) = 7(\frac{1}{1+3})x^{1+3} + C = \frac{7}{4}x^4 + C$.

2. The antiderivative is given by $G(x) = 3(\sin x) - 2(-\cos x) + C = 3\sin x + 2\cos x + C$.

3. The antiderivative is given by $R(t) = -5(e^t) + 4(\sec t) + C = -5e^t + 4\sec t + C$.

4. We first rewrite the function as $f(x) = 6x^{1/2} - 9x{-}2$. The antiderivative is given by

$F(x) = 6(\frac{1}{(1/2)+1})x^{(1/2)+1} - 9(\frac{1}{-2+1})x^{-2+1} + C = 4x^{3/2} + 9x^{-1} + C$.

5. We first rewrite the function as $h(t) = 4[\frac{1}{2}(1 + \cos t)]$. The antiderivative is given by

$H(t) = 4[\frac{1}{2}(t + \sin t)] + C = 2t + 2\sin t + C$.

6. We first rewrite the function as $g(x) = \frac{2x^2}{x^3} - \frac{7}{x^3} = 2x^{-1} - 7x^{-3}$. The antiderivative is given by

$G(x) = 2(\ln x) - 7(\frac{1}{-3+1})x^{-3+1} + C = 2\ln x + \frac{7}{2}x^{-2} + C$.

7. We first rewrite the function as $h(x) = x^3 + 6x^2 + 12x + 8$. The antiderivative is given by

$H(x) = (\frac{1}{3+1})x^{3+1} + 6(\frac{1}{2+1})x^{2+1} + 12(\frac{1}{1+1})x^{1+1} + 8(x) + C = \frac{1}{4}x^4 + 2x^3 + 6x^2 + 8x + C$.

8. The antiderivative is given by $S(x) = -5\tan^{-1} x$.

9. We find $f'(x) = 6(\frac{1}{2+1})x^{2+1} + 12(\frac{1}{1+1})x^{1+1} + 5(x) + C = 2x^3 + 6x^2 + 5x + C$, and

$f(x) = 2(\frac{1}{3+1})x^{3+1} + 6(\frac{1}{2+1})x^{2+1} + 5(\frac{1}{1+1})x^{1+1} + C(x) + K = \frac{1}{2}x^4 + 2x^3 + \frac{5}{2}x^2 + Cx + K$.

Using the fact $f'(0) = -1$ gives $-1 = 2(0)^3 + 6(0)^2 + 5(0) + C = C$. Using $f(0) = 1$ gives

$1 = \frac{1}{2}(0)^4 + 2(0)^3 + \frac{5}{2}(0)^2 + C(0) + K = K$. Thus $f(x) = \frac{1}{2}x^4 + 2x^3 + \frac{5}{2}x^2 - x + 1$.

10. We find $f'(x) = -\cos x - \sin x + C$ and $f(x) = -\sin x + \cos x + Cx + K$. Using $f'(\pi) = 1$

gives $1 = -\cos\pi - \sin\pi + C = -(-1) - 0 + C = 1 + C$ so that $C = 0$. Using $f(\pi) = \pi$

gives $\pi = -\sin\pi + \cos\pi + (0)(\pi) + K = -(0) + (-1) + (0) + K = K - 1$; hence $K = (\pi + 1)$

and $f(x) = -\sin x + \cos x + \pi + 1$.

11. The width of each rectangle is given by $\Delta x = \frac{8-0}{4} = 2$ and the interval $[0, 8]$ is divided into the four subintervals $[0, 2], [2, 4], [4, 6]$, and $[6, 8]$; the midpoints of the subintervals are 1, 3, 5, and 7. Hence we approximate the area by

$$A \approx f(1)\Delta x + f(3)\Delta x + f(5)\Delta x + f(7)\Delta x = (3)(2) + (9)(2) + (15)(2) + (21)(2) = 96.$$

The area under the curve is the area of the triangle with vertives (0,0), (8,0), and (8,24) so that the triangle has a base of 8 and height of 24 and therefore $A = \frac{1}{2}bh = \frac{1}{2}(8)(24) = 96$.

12. The width of each rectangle is given by $\Delta x = \frac{15-(-5)}{5} = 4$ and the interval $[-5, 15]$ is divided into the five subintervals $[-5, -1], [-1, 3], [3, 7], [7, 11]$, and $[11, 15]$; the left endpoints of the subintervals are -5, -1, 3, 7, and 11. Hence we approximate the area by

$$A \approx f(-5)\Delta x + f(-1)\Delta x + f(3)\Delta x + f(7)\Delta x + f(11)\Delta x = (26)(4) + (2)(4) + (10)(4) + (50)(4) + (122)(4) = 840.$$

For the second approximation we note that the right endpoints of the subintervals are given by -1, 3, 7, 11, and 15. Hence we approximate the area by

$$A \approx f(-1)\Delta x + f(3)\Delta x + f(7)\Delta x + f(11)\Delta x + f(15)\Delta x = (2)(4) + (10)(4) + (50)(4) + (122)(4) + (226)(4) = 1640.$$