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Preface

This book addresses several of the foundational problems in thermophysics, i.e. thermodynamics and statistical mechanics. It is an interdisciplinary work in that it examines the philosophical underpinning of scientific models and theories; it also refines the analysis of the problems at hand and delineates the place occupied by various scientific models in a generalized philosophical landscape.

Hence, our philosophical – or *theoretical* – inquiry focuses sharply on the concept of model; and our empirical – or *laboratory* – evidences are sought in the model building activities of scientists who have tried to confront the epistemological problems arising in the thermophysical sciences.

Primarily for researchers and students in physics, philosophy of science, and mathematics, our book aims at informing the Readers – with all the indispensable technical details made readily available – about the nature of the foundational problems, how these problems are approached with the help of various mathematical models, and what the philosophical implications of such models and approaches involve. Some familiarity with elementary thermophysics and/or with introductory level philosophy of science may help, but neither is a prerequisite. The logical and mathematical background required for the book are introduced in the Appendices. Upon using the Subject index, the Readers may easily locate the concepts and theorems needed for the understanding of every part of the book. A Name index lists the authors of the contributions we discuss with some detail.

Since ours is not a textbook in statistical mechanics, Readers should not expect the usual comprehensive coverage of standard textbook topics; for instance, they will not find any discussion of topics such as the Onsager relations or computer assisted proof, neither of which would inform our discussion of the roles of models.

We especially confront a pervasive trend of today's philosophical discussions of contemporary scientific theories: rather than *telling* what the theories amount to, we *demonstrate* how they are constructed and why they should be believed. We let our Readers see in detail the rigor of the results in thermophysics; how such rigor is achieved and what it signifies. By so doing, we may enable the philosophers of science to form independent judgment on foundational problems in thermophysics.

We also alert and inform the scientists concerning the philosophical issues and the nature of the reasoning pertinent to the praxis of thermophysics; some of the stones uncovered by our analysis are also offered as building materials to wider syntheses in natural science.

We eschew a rather widespread temptation in the science literature on modeling or mathematical modeling to identify these concepts with a listing of specific procedures or techniques in problem solving, however extensive the later may be.

Since the material in this book and its intended audience are diverse, a few words on how the book may be used are in order.

For physicists and mathematicians who are familiar with the standard presentation of thermophysics but are curious about the history and the fundamentals of probability and probabilistic reasoning, Chap. 4 may provide an interesting and useful point of entry. There we first summarize dual traditions in the interpretation of the concept of probability and then give a detailed study of some of the milestones from Bernoulli's binomial distribution and Pascal's triangle to Laplace's systematic rationalizations. We include also some historical details that illustrate the meaning of the theorems; meanwhile, these illustrations deepen our understanding of probability and provide an appreciation of their implications. We then show in Chap. 5 how a clear separation between the syntax and the semantics – which one can already witness in Hilbert's work in geometry – made it possible for Kolmogorov to systematize the formal aspects of probability. The importance of such a separation can be appreciated further when one gets to Chap. 6, where our discussion of the competing semantics shows that waiting for a settlement of the semantics for probability would have greatly hampered the understanding of its formal properties. Our examination of the notion of randomness in Chaps. 5 and 6 (and the references cited and briefly introduced therein) should well inform those who are curious about the foundations of probability theories.

If our Readers from physics and mathematics are, moreover, curious about the philosophical implications of the separation of syntax and semantics of a theory, and about how the conception of scientific theories is analyzed in the context of such a separation, they will find in Chap. 1 an in-depth discussion of the philosophical landscape. Our Readers should look there for the identification of the components involved in building a bridge across the gap between the logician's (or philosopher's) conception of models and that of scientists'; see also Appendix E. In particular, we explain how the recognition of the syntax/semantics distinction helps recognize general features of theory construction in thermophysics.

For physicists and mathematicians who have a working knowledge of thermophysics but are concerned with the validity of statistical mechanics, Chap. 7 is the place to start reading. We articulate first in this chapter the original hypothesis of ergodicity by Boltzmann and evaluates what Boltzmann could

only have meant by it in the light of the limitations imposed by his finitist philosophical position, and by the contemporary mathematics (the absence of properly formulated theories of measure and integration) and physics (the scarcity of worked-out models beyond the ideal gas). We then discuss two improved versions of ergodicity: Birkhoff's and von Neumann's ergodic theorems, which rendered the notion of ergodicity mathematically rigorous but made its application to real physical systems a problem that is still with us today. Chaps. 8 and 9 examine finer properties and models of the ergodic hierarchy. An in-depth critical discussion of the relevance of several models to the foundations of statistical mechanics is given there as well. One of our purposes is to provide specific anchors for the discussion of the deliberately controversial assertion according to which ergodic conditions are neither necessary nor sufficient for a proper understanding of the physical theory.

Students of formal logic will find that their field enters in at least three ways in our enterprise: (1) in the sustained distinction between the properties of language (syntax) and its interpretations (semantics); (2) in the discussion of the very notion of model; (3) in the use of recursive function theory and algorithmic complexity to discuss randomness and, consequently, entropy.

For philosophers of science in general, and of physics in particular, this is a book about models and model-building in physics. For those who are familiar with the literature of the semantic (or structural or architectonic) view of scientific theories, Chap. 1 offers a critique of the existing views and a proposal for a new, hybrid view. The problem was flagged in the title quotation from van Fraassen and the chapter endeavors to address that problem. For those who are not familiar with this literature, the chapter will serve as an introduction to a general philosophical account of models and theories in science; these Readers will find here a list of different types of theories and models, and then how these fit into a general scheme of theory construction and testing in the praxis of science. Chaps. 2 and 3 are introductory materials for thermophysics. Chap. 2, *Thermostatistics*, not only informs our Readers about the essentials of a phenomenological theory of macroscopic systems but also demonstrates how the separation of syntax and semantics works for the construction and analysis of such a theory. Chap. 3 provides a simple but thoroughly worked-out tour of some of the most significant microscopic models of gases; see also Chap. 11.

Those philosophers of physics who are anxious to examine some real models in physics – and also to see how these models are conceived and used – may want to go directly to the Chaps. 7 to 15. The models in Chaps. 7 to and 10 are all directed towards the foundations of thermophysics; and the models in Chaps. 11 to 15 are mostly chosen for their delivering microscopic explanations for well tested thermo-phenomena. For those Readers who are familiar with the philosophical literature on foundations of statistical mechanics and problems of ergodicity, a wealth of carefully worked out models is presented which exhibits different properties on the ergodic hierarchy. These Readers

will find out not only what these models mean to the foundations of statistical mechanics but also why they are so meant. Enough technical steps are detailed to allow an informed appreciation of the mathematical workings of the models. Specific applications of the formalism of classical and quantum statistical mechanics are the focus of Chaps. 11 to 14; we do not shy from showing step-by-step to our Readers how the complex phenomena of phase transition and of criticality are explained in (classical) lattice models via some of the intricate and powerful tools that mathematics offers to physics. The renormalization semi-group method is one of them. In Chap. 15, we illustrate with the help of specific (quantum) models some of the questions non-equilibrium statistical mechanics has begun to answer.

A most prominent feature of model-building is the relative places of idealization and approximation. Throughout the whole book we have laid emphasis on the various manifestations of these two closely related concepts. We give an integrated summary of such discussions in the last chapter of the book: Chap. 16. Since the entire range of applications of these two concepts in science is relatively understudied in the philosophy of science literature, part of our intention for that chapter is to stimulate further philosophical investigations of these concepts. Simulation is another understudied concept in philosophy, although it is becoming more and more important in scientific research. We give a brief account of various aspects of the concept, however without pretending to do it full justice. Nevertheless, our intention – here also – is to stimulate further studies of the concepts in connection with the practice of model-building and model-testing in physical sciences.

In most of our book, the microscopic world is viewed through the lenses of classical rather than quantum mechanics. This is done for two reasons. The first is that we believe that the main problem posed by the attempt to reconcile macro- and micro-scopic theories is a reduction process involving the interplay of various scales; and that the understanding of this interplay is not primarily dependent on whether the microscopic picture is classical or quantum. In this context, our Readers steeped in the traditional philosophic discourse may want to pay attention to the issues involved in the pragmatic meaning of the mathematical notion of limit; see e.g. Sects. 1.6 and 16.4. These issues are brought forward specifically in connection with the use of (1) the thermodynamical limit (described Sects. 10.2–12.1. and implemented, for instance in Sects. 12.2 and the Models 2, 3, 4 of Sect. 15.3; see also its role in Chap. 14; (2) the high temperature limit (Sect. 10.3); (3) the critical scaling in phase transitions (Chap. 13); and (4) the van Hove limit (described in Sect. 15.2 and illustrated in detail with Models 3 and 4 of Sect. 15.3). These limits are presented here in a manner that ought to illuminate such reflections.

The second reason behind our taking the classical rather than the quantum option is that we want to avoid the danger that the conceptual problems involved in the reduction be masked by the conceptual difficulties inherent to a fully consistent presentation of quantum premises. Nevertheless, we do

consider explicitly quantum situations: in the end of Sect. 5.3; in the course of Sect. 6.3; in several aspects of Chap. 8; in Sects. 10.2; and in Chaps. 14 and 15.

Finally, we should mention that we systematically verified – and modified when necessary – the translations of all the original texts we quote.

Written and oral comments on various parts of the book, as well as discussions on related materials, are gratefully acknowledged. We were indeed privileged to engage in stimulating conversations with several colleagues, among whom we wish to mention the following:

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Antoinette Emch-Dériaz read through the typescript, some parts several times over, shooting down obscure sentences and typographical errors; yet, the responsibility is ours for those that escaped the hunt. She and Mingmin Zhu cannot be thanked enough for their unfailing patience and steadfast support.

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