Computable Riesz Representation for the Dual of C[0;1]

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By the Riesz representation theorem for the dual of $C[0;1]$, for every continuous linear operator $F : C[0;1] \to \mathbb{R}$ there is a function $g : [0;1] \to \mathbb{R}$ of bounded variation such that

$$F(f) = \int f\, dg \quad (f \in C[0;1])\,.$$

The function g can be normalized such that $V(g) = \|F\|$. In this paper we prove a computable version of this theorem. We use the framework of TTE, the representation approach to computable analysis, which allows to define natural computability for a variety of operators. We show that there are a computable operator S mapping g and an upper bound of its variation to F and a computable operator S' mapping F and its norm to some appropriate g.