

Random Continuous Functions

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We investigate notions of algorithmic randomness in the space $\mathcal{C}(\{0,1\}^{\mathbb{N}})$ of continuous functions on $\{0,1\}^{\mathbb{N}}$. A probability measure is given and a version of the Martin-Löf test for randomness is defined which allows us to define a class of (Martin-Löf) random continuous functions. We show that random Δ_2^0 continuous functions exist, but no computable function can be random. We show that a random function maps any computable real to a random real and that the image of a random continuous function is always a perfect set and hence uncountable. We show that for any $y \in \{0,1\}^{\mathbb{N}}$, there exists a random continuous function F with y in the image of F . Thus the image of a random continuous function need not be a random closed set.