Ninth Annual Ulam Lecture, New Vistas in Control of Complex Systems: Logical and geometric aspects of hybrid control and their application to real time systems

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Traditional linear and optimal control theory is not up to the task of extracting control for complex systems such as aircraft traffic control, military command and control, distributed enterprise systems. The problem is that these systems usually have many non-linearities in immense phase spaces and interact with discrete systems, which are logic, or rule- based systems and not amenable to traditional linear or optimal control technology. In 1991 DARPA held a meeting about this problem, a meeting at which I met Wolf Kohn (then a senior Boeing engineer). In response Kohn and I developed a theory of interacting systems of discrete digital programs and continuous devices. Our program was and is to develop methods for extracting digital control programs for such systems, which force them to obey their performance specifications. I call this the "Fundamental Problem of Hybrid Systems".

I named the area "Hybrid Systems", held four international conferences and published four volumes, and succeeded in fomenting a thriving worldwide community at the interface of logic, computer science, and control engineering. Over the past fourteen years Kohn and I have developed in many papers and in a commercial company our own method of modeling hybrid systems by Finsler manifolds. We represent each control problems as the problem of approximating to solutions to relaxed calculus of variations problems on a suitable Finsler manifold. The manifold incorporates both a continualized version of all discrete logical rules or constraints and all the continuous constraints given by ordinary differential equations, and a Lagrangian optimization function on trajectories to the desired goal. Optimal controls give optimal trajectories to the goal, which are Finsler Geodesics.

The main technology developed consists of algorithms for decomposing the manifold into small subregions and computing the approximately optimal control to be used in each subregion. Since almost all problems lead to computationally intractable relaxed optimal solutions, one does not ever compute controls leading to actually optimal trajectories. One only computes the change in control needed, based on the region you are in, to take yourself in the approximate direction of a geodesic to the goal. The finite control automaton dictating these shifts in control based on the region you are in is the origin of our finite automaton digital control. This is where the connection with logic arises.

These have to be real time algorithms, because as a real system experiences real dynamics, it has to change the decomposition and rules on the fly. It has taken a long time and a lot of capital to develop real time algorithms and software. Some commercial products based on this technology now exist.

I will sketch what is involved in some detail, and examine what future research directions in control theory and logic this work suggests.