

On Computable Compact Operators on Banach Spaces

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We develop some parts of the theory of compact operators from the point of view of computable analysis. While computable compact operators on Hilbert spaces are easy to understand, it turns out that these operators on Banach spaces are harder to handle. Classically, the theory of compact operators on Banach spaces is developed with the help of the non-constructive tool of sequential compactness. We demonstrate that a substantial amount of this theory can be developed computably on Banach spaces with computable Schauder bases that are well-behaved. The conditions imposed on the bases are such that they generalize the Hilbert space case. In particular, we prove that the space of compact operators on Banach spaces with monotone, computably shrinking and computable bases is a computable Banach space itself and operations such as composition with bounded linear operators from left are computable. Moreover, we provide a computable version of the Theorem of Schauder on adjoints in this framework and we discuss a non-uniform result on composition with bounded linear operators from right.