$FD(\omega)$ in Densely Embeddable in $\mathcal{P}_M$

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Let $\mathcal{P}_M$ be the lattice of degrees of non-empty Π^0_1 subsets of 2^ω under Medvedev reducibility. Binns and Simpson proved that $FD(\omega)$, the free distributive lattice on countably many generators, is lattice-embeddable below any non-zero element in $\mathcal{P}_M$. Cenzer and Hinman proved that $\mathcal{P}_M$ is dense, by adapting the Sacks Preservation and Sacks Coding strategies used in the proof of the density of the c.e. degrees. With a construction that is a modification of the one by Cenzer and Hinman, we improve upon the result of Binns and Simpson by showing that for any $\mathcal{P} <_M \mathcal{Q}$ in $\mathcal{P}_M$, we can lattice embed $FD(\omega)$ strictly between $\mathcal{P}$ and $\mathcal{Q}$. We also note that in our construction and in the one used by Cenzer and Hinman, all injury is finite, in contrast to the infinite injury in the construction for the Sacks Density Theorem.