Computability of Solutions of Operator Equations

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We study operator equations within the Turing machine based framework for comput ability in analysis. Is there an algorithm that maps pairs (T, u) (where T is given in form of a program) to good approximate solutions of $Tx = u$? Here we consider the case when T is a bounded linear mapping of Hilbert spaces. We are in particular interested in computing the *generalized inverse* $T^{\dagger}u$ which is the standard concept of solution in the theory of inverse problems. Ty pically, $T^{\dagger}$ is discontinuous (i.e. the equation $Tx = u$ is *ill-pose d*) and hence no computable mapping. However, we will use effective versions of theorems from the theory of *regularization* to show that the mapping $(T, T^{*}, u, \|T^{\dagger}u\|) \mapsto T^{\dagger}u$ is computable. We then go on to study the computability of *average-case solutions* with respect to Gaussian measur es which have been considered in *information based complexity.* Here $T^{\dagger}$ is considered as an element of an L^2-space. We define suitable representa tions for such spaces and use the results from the first part of the paper to sh ow that $(T, T^{*}, \|T^{\dagger}\|_{L^2}) \mapsto T^{\dagger}$ is computable.