

2010 SEALS Abstracts

Speaker: Paul Brodhead, Virginia State University

Title: Effective Capacity and Random Closed Sets

Abstract: I will discuss recent joint work, with Douglas Cenzer, investigating the connection between measure and capacity in the space of closed subsets of Cantor space. For any computable measure, a computable capacity may be defined by letting $T(Q)$ be the measure of the family of closed sets which have nonempty intersection with Q . We prove an effective version of the Choquet's theorem by showing that every computable capacity may be obtained from a computable measure in this way. Furthermore, we establish conditions which characterize when the capacity of a random closed set is nonnegative. Finally, we construct, for certain measures, an effectively closed set with positive capacity and with Lebesgue measure zero.

Speaker: Jennifer Chubb, University of San Francisco

Title: Algorithmic properties of orders on groups

Abstract: A countable group is computable if it is computable as a set and there is an algorithm for computing the group operation. A group is left-orderable (bi-orderable) when it admits a linear ordering of its elements that is left-invariant (bi-invariant) under the group action. In this talk, we will investigate algorithmic properties of orderings of computable groups and see different conditions that ensure a group admits an ordering of every possible degree of algorithmic complexity. This work is joint with Mieczyslaw Dabkowski and Valentina Harizanov.

Speaker: Alan Dow, University of North Carolina, Charlotte

Title: Normality and Cohen/Random reals

Abstract: The consistency of the Normal Moore Space Conjecture relied heavily on the fact that adding either Cohen or Random reals preserved when sets could not be separated by open sets in a topological space. In response to a question from the Open Problems in Topology II by Ohta and Yamazaki, we investigate preservation questions about separating sets by continuous real-valued functions.

Speaker: Damir Dzhafarov, University of Chicago

Title: Stable Ramsey's Theorem and Measure

Abstract: The stable Ramsey's theorem for pairs has been the subject of numerous investigations in mathematical logic. We introduce a weaker form of it by restricting from the class of all stable colorings to subclasses of it that are non-null in a certain effective measure-theoretic sense. We show that uniformity results for the two principles agree below $\mathbf{0}'$ but are not the same in general. We also answer the analogs of two well known questions about the stable Ramsey's theorem by showing that our weaker principle does not imply COH or WKL_0 in the context of reverse mathematics.

Speaker: Johanna Franklin, University of Waterloo, Canada

Title: Randomness and the Ershov hierarchy

Abstract: To determine whether a real is Martin-Loef random, we use Martin-Loef tests: uniform sequences of r.e. subsets of the Cantor space whose measures converge uniformly to 0. In this talk, I will define new notions of randomness in which the subsets of the Cantor space are n -r.e. for some n greater than 1. I will present various ways in which a test can be considered to be n -r.e. and describe the classes of n -r.e. random reals. In one case, the resulting class of random reals is a proper subclass of the Martin-Loef random reals but properly contains the Demuth random reals and the weakly 2-random reals.

Speaker: Valentina Harizanov, George Washington University

Title: Degree spectra and the jump hierarchy

We will present results on Turing degree spectra of structures and relations in the context of jump hierarchy. More specifically, we investigate when such spectra are the $high_n$ or the $nonlow_n$ degrees, where n is a natural number. In particular, we are interested in the spectra of linear orderings, as well as spectra of relations on the ordered set of rationals. This is joint work with Frolov, Kalimullin, Kudinov, and R. Miller.

Speaker: Denis Hirschfeldt, University of Chicago

Title: Reverse Mathematics of Model Theoretic Principles

Abstract: I will discuss how tools from reverse mathematics and computability theory can be used to establish close connections between basic model theoretic results, combinatorial principles, and notions of genericity. In particular, I will discuss joint work with Shore and Slaman on the Atomic Model Theorem (the fact that every complete atomic theory in a countable language has a countable atomic model), and with Lange and Shore on the Homogeneous Model Theorem (a characterization of the sets of types of countable theories that are type spectra of homogeneous models).

Speaker: Bart Kastermans, University of Colorado

Title: Formalizing Set Theory; on a simple example.

Abstract: We all know the vast difference between the notion of proof as described in an introductory logic class, and the one we use on a daily basis when doing mathematics. The simple notion of proof has a large benefit that it is easy to check. Ideally all our proofs would be given with this much detail (next to the intuitive explanation) so that a higher degree of certainty can be obtained. In practice this is undoable, or at least highly unpleasant, to do by hand. To facilitate this certain computer systems have been developed to help with this. In this talk I'll give a short introduction to such a system, and show how I have used it to formalize a simple example of a cofinitary group.

Speaker: Geoff Laforte, University of West Florida

Title: Equivalence structures and isomorphisms in the difference hierarchy

Abstract: Effective categoricity can be examined via isomorphisms available in Ershov's difference hierarchy and in related hierarchies. There are various possibilities here for the notion of isomorphism and some interesting results involving even structures as simple as equivalence relations. (This is joint work with Doug Cenzer and Jeff Remmel.)

Speaker: Julia Knight, Notre Dame University

Title: Describing free groups

Abstract: Sela showed that the free groups on more than one generator are all elementarily equivalent. There is related work by Karlampovich and Myasnikov. Jacob Carson, Valentina Harizanov, Karen Lange, Christina Maher, Charles McCoy, Andrei Morozov, Sara Quinn, John Wallbaum and I looked for the simplest possible infinitary descriptions of the different free groups. To show that our descriptions are optimal, we used index sets. We obtained sharp results for the free groups of finite rank. For the group of infinite rank, we showed that there is a computable Π_4 description, but we could not show that this is optimal. Wallbaum and McCoy continued working, and succeeded in doing this.

We also considered the problem of finding a basis for a given copy G of the free group of infinite rank. The large group of authors succeeded in showing that there is a basis that is $\Pi_2^0(G)$. Wallbaum and McCoy showed that this is best possible.

Speaker: Tamas Matrai, Rutgers University

Title : On F_σ p -ideals and Tukey reducibility

Abstract: The structure of all subsets of natural numbers with almost inclusion embeds into the family of F_σ p -ideals partially ordered by Tukey reducibility. I relate this result to previously known cofinal diversity results and Banach space constructions.

Speaker: Itay Neeman, UCLA

Title: Determinacy

Abstract: Considerations that involve infinite games come up in engineering and economics. Typically a particular round or a sequence of rounds is repeated indefinitely, with end payoff obtained by some long-term averaging, or by more complicated formulas that require particular states to be avoided, and other states to occur infinitely many times. We begin by discussing these games and the various notions of strategies and equilibria associated to them. From there we will be led gradually back in time, to principles of determinacy—asserting the existence of winning strategies in perfect information, infinite, zero sum games—and their use by descriptive set theorists studying the classical uniformization problem for definable sets of reals. The talk will be self-contained and accessible to a general audience.

Speaker: Selwyn Ng, University of Wisconsin

Title: Jump inversions inside effectively closed sets and applications to randomness

Abstract: The jump operator that takes a set X to the Halting problem relative to X is one of the most well-studied notion in computability theory. We will talk about inversions of the jump operator on Π_1^0 classes. These have implications to the study of the jump operator on various notions of effective randomness. For instance we will characterize the jumps of weakly 2-random sets which are not 2-random. We also discuss various techniques for coding information into certain incomplete Martin-Lof randoms through Π_1^0 classes.

Speaker: Dilip Raghavan, University of Toronto, Canada

Title: A model with no strongly separable MAD families

Abstract: We answer a question of Shelah and Steprāns (ShSr:931) by producing a model of ZFC where there are no strongly separable almost disjoint families. The notion of a strongly separable almost disjoint family is a natural strengthening of the well known notion of a completely separable almost disjoint family that is connected to the metrizable problem for countable Fréchet groups.

Speaker: Jan Reimann, UC Berkeley

Title: Randomness and Definability

Abstract: The duality between measures and the sets they "charge" is a central theme in modern analysis. An effective analogue of this question is: Given a real X , does there exist a (probability) measure relative to which X is effectively random in the sense of Martin-Loef (so that X is not an atom of the measure)? It turns out the view from logic opens up a new perspective stemming from the interplay between randomness and definability strength. I will try to present various aspects of this relation. These will include determinacy, the use of higher infinities, rank functions for coanalytic sets, Hausdorff measures and potential theory.

Speaker: Jeff Remmel, UC San Diego

Title: Σ_1^0 and Π_1^0 structures

Abstract: We study computability theoretic properties of Σ_1^0 and Π_1^0 equivalence structures and how they differ from computable equivalence structures or equivalence structures that belong to the Ershov difference hierarchy. Our investigation includes the complexity of isomorphisms between Σ_1^0 equivalence structures and between Π_1^0 equivalence structures.

Speaker: Grigor Sargysan, UCLA

Title: On the strength of the failure of square at a singular strong limit cardinal.

Abstract: Jensen in 1970's showed that the failure of his square principle at a singular strong limit cardinal implies that $0^\#$ exists, and hence, V is not equal to L . The consistency strength of failure of square at a singular strong limit cardinal has been since a topic that fascinated many inner model theorists. In this talk we will outline a recent improvement of a result of Steel. In particular, we will try to explain some ideas involved in the proof of the following theorem.

Theorem (S.) Suppose $\kappa > \aleph_{\omega}$ is a singular strong limit cardinal such that \square_{κ} fails. Then there is a non-tame mouse.

Speaker: Mariya Soskova, Sofia, Bulgaria

Title: Characterizing the strength of the local theory of the enumeration degrees

Abstract: In this joint project with Hristo Ganchev we prove that the the first order theory of Peano arithmetic can be interpreted in the local theory of the enumeration degrees. We use the coding of standard models of arithmetic method introduced by Nies, Shore and Slaman. Our main coding tool is the notion of a K -pair, introduced and used by Kalimullin to show that the enumeration jump is definable.

Speaker: Simon Thomas, Rutgers University

Title: Borel Determinacy and the Word Problem for Finitely Generated Groups

Abstract: It is well-known that for each $A \in 2^{\mathbb{N}}$, there exists a finitely generated group G_A such that the word problem for G_A is Turing equivalent to A . In this talk, we show that there does not exist a Borel choice of G_A such that if $A \equiv_T B$, then $G_A \cong G_B$.

Speaker: Ferit Toska, University of Florida

Title: Computability of Countable Subshifts

Abstract: The computability of countable subshifts and their members is examined. Results include the following. Subshifts of Cantor-Bendixson rank one contain only eventually periodic elements. Any rank one subshift, in which every limit point is periodic, is decidable. Subshifts of rank two may contain members of arbitrary Turing degree. In contrast, effectively closed (Π_1^0) subshifts of rank two contain only computable elements, but Π_1^0 subshifts of rank three may contain members of arbitrary c. e. degree. There is no subshift of rank ω .

Speaker: Rebecca Weber, Dartmouth College

Title: Reals that are low for information

Abstract: Kolmogorov complexity measures the information content of a finite string. We define the mutual information of two finite strings as the sum of their individual complexities minus the complexity of their encoding as a pair. When the strings are closely related, the complexity of the pair will be smaller and hence the mutual information larger. For infinite sequences there is no universally agreed-upon definition for mutual information. I'll present one contender and argue for its plausibility, and discuss ongoing work with Denis Hirschfeldt on sequences that are low for information: those that have finite mutual information with themselves. These have interesting relationships with K -triviality and other properties of computational weakness.

Speaker: Guohua Wu, Nanyang University, Singapore

Title: Separating cuppable degrees from low_n -cuppables

Abstract: In 2000, Li, Wu and Zhang gave a hierarchy of cuppable degrees: a c.e. degree \mathbf{a} is low_n -cuppable ($n > 0$), if there is a low_n c.e. degree cupping a to $\mathbf{0}'$, and proved that there is a degree low_2 cuppable, but not low-cuppable. In this talk, I will present a recent result (joint work with Greenberg and Ng) about the existence of a cuppable degree, but not low_n cuppable for any n .

Speaker: Beatriz Zamora-Aviles, York University, Canada

Title: Structure of order ideals

Abstract: Let $BH1$ be the set of bounded positive operators of norm at most one on an infinite dimensional Hilbert space. We give a definition of analytic P -ideals on this space and give a characterization of them. This characterization is a non-commutative version of the well-known characterization of analytic P -ideals on omega due to S. Solecki. We also obtain a characterization of certain meager subsets of this space. This is a non-commutative analog to Jalali-Naini and Talagrand characterization of meager subsets of the cantor space.