

MTG 6401 Supplemental Homework

Any problems are due in hard copy form on Wednesday, December 14, 5PM, in my office (slide under door). Each problem is worth 5 points except the last which is worth 10 points.

1. The Contraction Mapping Theorem: If (X, d) is a complete metric space and $f : X \rightarrow X$ satisfies for some $0 < k < 1$ and for all $x, y \in X$

$$d(f(x), f(y)) \leq k d(x, y),$$

then f has a unique fixed point P , and for any point $x \in X$, $f^n(x) \rightarrow P$ as $n \rightarrow \infty$.

2. Let (Σ_A^+, σ) be the subshift of finite type defined by the transition matrix A with $A^k > 0$ for some $k > 0$. Show that (Σ_A^+, σ) has sensitive dependence on initial data (sdid) everywhere.

3. Find the fixed points of this DE and classify their stability type

$$\begin{aligned}\dot{x} &= x^2 - y - 1 \\ \dot{y} &= y^2 - 3y(x + 1).\end{aligned}$$

4. Prove the one-dimensional Brouwer fixed point theorem: If $f : [0, 1] \rightarrow [0, 1]$ is continuous, then it has a fixed point.

5. If $f : S^1 \rightarrow S^1$ is **orientation reversing**, show that f has at least two fixed points.

6. A homeomorphism between two metric spaces $h : (X, d_X) \rightarrow (Y, d_Y)$ is called *bi-Lipschitz* if there is a constant K so that for all $x_1, x_2 \in X$ and $y_1, y_2 \in Y$,

$$d_Y(h(x_1), h(x_2)) \leq K d_X(x_1, x_2) \text{ and } d_X(h^{-1}(y_1), h^{-1}(y_2)) \leq K d_Y(y_1, y_2).$$

Prove that if $f : X \rightarrow X$ and $g : Y \rightarrow Y$ are conjugate by a bi-Lipschitz homeomorphism $h : (X, d_X) \rightarrow (Y, d_Y)$ (i.e. $h \circ f = g \circ h$) and f has sdid everywhere, then so does g .

7. Do one of the tutorials at <http://www.dynamicalsystems.org/tu/tu/> and write a one page essay on what you learned. This has to be all original, not cut and pasted from the tutorial!