

# Contents

Preface	vii
<b>I. Basic Methods</b>	
<b>Chapter 1</b>	
<b>Seven is More than Six. The Pigeon- Hole Principle</b>	<b>1</b>
Exercises . . . . .	9
Supplementary Exercises . . . . .	11
Solutions to Exercises . . . . .	11
<b>Chapter 2</b>	
<b>One Step at a Time. The Method of Mathematical Induction</b>	<b>19</b>
2.1 Weak Induction . . . . .	19
2.2 Strong Induction . . . . .	25
Exercises . . . . .	26
Supplementary Exercises . . . . .	28
Solutions to Exercises . . . . .	29
<b>II. Enumerative Combinatorics</b>	
<b>Chapter 3</b>	
<b>There Are A Lot Of Them. Elemen- tary counting problems</b>	<b>37</b>
3.1 Permutations . . . . .	37
3.2 Strings over a finite alphabet . . . . .	40

3.3 Choice problems . . . . .	43
Exercises . . . . .	47
Supplementary Exercises . . . . .	51
Solutions to Exercises . . . . .	52
<b>Chapter 4 No matter how you slice it. The bi-</b>	
<b>nomial theorem and related identities</b>	<b>65</b>
4.1 The binomial theorem . . . . .	65
4.2 The multinomial theorem . . . . .	70
4.3 When the exponent is not a positive integer . . . . .	73
Exercises . . . . .	74
Supplementary Exercises . . . . .	78
Solutions to Exercises . . . . .	78
<b>Chapter 5 Divide and Conquer. Partitions.</b>	<b>89</b>
5.1 Compositions . . . . .	89
5.2 Set partitions . . . . .	91
5.3 Integer partitions . . . . .	94
Exercises . . . . .	100
Supplementary Exercises . . . . .	102
Solutions to Exercises . . . . .	102
<b>Chapter 6 Not So Vicious Cycles. Cycles in Permutations</b>	<b>109</b>
6.1 Cycles in Permutations . . . . .	110
6.2 Permutations with restricted cycle structure . . . . .	116
Exercises . . . . .	120
Supplementary Exercises . . . . .	122
Solutions to Exercises . . . . .	123
<b>Chapter 7 You Shall Not Overcount. The sieve.</b>	<b>131</b>
Exercises . . . . .	138
Supplementary Exercises . . . . .	139
Solutions to Exercises . . . . .	139
<b>Chapter 8 A Function Is Worth Many Numbers.</b>	
<b>Generating Functions</b>	<b>145</b>
8.1 Ordinary generating functions . . . . .	145
8.1.1 Recursive formulae and generating functions . . . . .	145
8.1.2 Products of generating functions . . . . .	152

8.1.3	Compositions of Generating Functions . . . . .	156
8.2	Exponential Generating Functions . . . . .	160
8.2.1	Recursive Formulae and Exponential Generating Functions	160
8.2.2	Products of exponential generating functions . . . . .	162
8.2.3	Compositions of Exponential Generating Functions . . . . .	165
	Exercises . . . . .	168
	Supplementary Exercises . . . . .	170
	Solutions to Exercises . . . . .	171

### III. Graph Theory

<b>Chapter 9</b>	<b>Dots and Lines. The origins of graph theory</b>	<b>183</b>
9.1	The notion of graphs. Eulerian walks . . . . .	184
9.2	Hamiltonian Cycles . . . . .	188
9.3	Directed graphs . . . . .	190
9.4	The notion of Isomorphisms . . . . .	193
	Exercises . . . . .	196
	Supplementary Exercises . . . . .	200
	Solutions to Exercises . . . . .	200
<b>Chapter 10</b>	<b>Staying Connected. Trees</b>	<b>207</b>
10.1	Minimally connected graphs . . . . .	207
10.2	Minimum-weight spanning trees . . . . .	214
10.3	Graphs and matrices . . . . .	218
10.3.1	Adjacency matrices of graphs . . . . .	218
10.4	The number of spanning trees of a graph . . . . .	221
	Exercises . . . . .	226
	Supplementary Exercises . . . . .	229
	Solutions to Exercises . . . . .	230
<b>Chapter 11</b>	<b>Finding a good match. Coloring and Matching</b>	<b>239</b>
11.1	Introduction . . . . .	239
11.2	Bipartite graphs . . . . .	241
11.3	Matchings in bipartite graphs . . . . .	246
11.4	More Than Two Colors . . . . .	252
11.5	Matchings in Graphs that are not Bipartite . . . . .	254
	Exercises . . . . .	258



*Contents*

v

15.4.2 Existence proofs using Expectation . . . . .	354
15.4.3 Conditional Expectation . . . . .	356
Exercises . . . . .	358
Supplementary Exercises . . . . .	360
Solutions to Exercises . . . . .	362
<b>Chapter 16 Partial orders and lattices</b>	<b>369</b>
16.1 The Notion of Partially Ordered Sets . . . . .	369
16.2 The Möbius function of a poset . . . . .	374
16.3 Lattices . . . . .	383
Exercises . . . . .	390
Supplementary Exercises . . . . .	391
Solutions to Exercises . . . . .	393
Bibliography	401
Index	403