

Aristotle on the Potentially Infinite

Marija Jankovic (2005)

Department of Philosophy

As Charlton (1991, p.129) notes, to understand a thinker's finitism it is important to understand what kinds of infinity she rejects. Moreover, as I shall argue, to understand Aristotle's finitism, it is central to understand not only what kinds of infinity he rejects but also on what grounds he rejects them, for that will allow us to recognize some subtleties of Aristotle's account that would otherwise remain unnoticed. In this paper, I attempt to give a systematic account of Aristotle's notion of the potentially infinite by closely scrutinizing *Physics* III 6. In part 1, I discuss Aristotle's argument that the infinite exist only potentially. In part 2, I argue that the distinction between infinite processes and the products of these processes is also relevant for understanding Aristotle's views on the existence of the infinite.

1. Existence and non-existence of the infinite

After arguing in *Physics* III 5 that there cannot exist an infinitely extended body, Aristotle notes that we cannot deny the infinite any existence at all, for that would lead to some impossible consequences, namely "there will be a beginning and an end of time, a magnitude will not be divisible into magnitudes, number will not be infinite" (*Phys.* III 6, 206a10-12). It is important to draw attention to this stretch of text, for it gives us a clue about what Aristotle is taking for granted in *Phys.* III 6. Since my primary concern in this paper is the infinite in respect of a magnitude, let us note that straight away, Aristotle is stating that the consequence that a

magnitude is not divisible into magnitudes¹ is enough for a *modus tollens* of a view (it is an “impossible consequence”).

It seems clear that Aristotle holds that there must be a sense in which the infinite does exist, and further down (206a18), that it exists *potentially*. That the infinite exist potentially is presented as a conclusion of the following curious argument, which takes the apparent form of a disjunctive syllogism.

[1] Now things are said to exist both potentially and in fulfillment. [2] Further, a thing is infinite either by addition or by division. [3] Now, as we have seen, magnitude is not actually infinite. [4] But by division it is infinite. ([4*]For there is no difficulty in refuting the theory of indivisible lines.) [5] The alternative then remains that the infinite has a potential existence. (206a14-18)

It is difficult to see how to make Aristotle’s argument in this passage valid. If we take into account only disjunctive premises ([1] and [2]), there seem to be four possible ways in which the infinite may exist: (i) potential infinite by addition, (ii) actual infinite by addition, (iii) potential infinite by division, (iv) actual infinite by division. Normally, in a disjunctive syllogism one would expect categorical premises to exclude all of the inadmissible options; however, categorical premises of this argument do not appear to do so. In [3] Aristotle states that the actual infinite does not exist and justifies it by invoking previous discussion (“*as we have seen*”). Thus, it would seem that Aristotle is excluding options (ii) and (iv). Still, so far he has not said anything about the infinite by division. What we have seen, namely in III 5, is that there cannot be an *infinitely extended body*. Strangely enough, the next categorical premise [4] contrasts this sense of the infinite with the infinite by division *simpliciter*. (“But by division it is infinite.”)

¹ However, note that Aristotle has not yet qualified a way in which a magnitude is divisible into magnitudes.

This would seem to leave both option (iii) and option (iv) still open. The rationale for this premise is *that there is no difficulty in refuting the theory of indivisible lines*. Bostock (1973, p.38) too, notes that Aristotle does not offer an argument that a magnitude is not actually infinite by division and claims that “this is simply a lacuna in Aristotle’s account, and a lacuna of some importance [...]”.

Thus, Aristotle’s argument seems clearly invalid. However, as I have suggested earlier, a closer look at the justification of categorical premises will provide us with a better understanding of what exactly is going on in this passage. It turns out that what might be missing from *Phys.* III 6, can be found elsewhere in Aristotle’s writings, and that there is a reasonable explanation why Aristotle would not go into the details of that argument here. Therefore, let us closely examine both categorical premises.

1.1 There is no infinitely large magnitude

In the first categorical premise Aristotle claims that “magnitude is not actually infinite”, and refers us back to *Phys.* III 5 for the justification. This certainly is a curious way of formulating the conclusion of III 5 – that there cannot be an infinitely large body. We need not go into details of Aristotle’s argument in III 5 here², but returning to the disjunctive syllogism of III 6 it is now clear, when we take Aristotle’s justification into account, that premise [3] should be read as claiming that there is no actually infinite *by addition*, since there is no infinite magnitude.

² Aristotle’s rejection of the infinitely large body is based on physical considerations of the nature of elements and motion.

This leaves open the question whether there is also a potentially infinite by addition, which I will discuss once we have gained a clearer understanding of the sense in which “potential” figures in Aristotle’s discussion.

1.2 But by division a magnitude is infinite

Certainly, by III 6, Aristotle has not offered any arguments for this premise. As we have already mentioned, the infinite divisibility of a magnitude is a part of preliminary knowledge Aristotle is assuming his readers are bringing into their understanding of the chapter. However, to make Aristotle’s argument valid, we need *not only* the argument that a magnitude is infinitely divisible, but also the argument that *it is so only potentially*.

We turn to *On Generation and Corruption (GC) I 2* for some clues. Here (*GC I 2*, 316a15-317a18), Aristotle discusses the problems which arise if we suppose that a magnitude had been divided “through and through”. “What, then, will remain?”, Aristotle asks. Not a magnitude, since that would contradict the assumption that all the divisions had been made. Not points though, since “if it [i.e. a body] consists of points, it will not possess any magnitude. For when the points were in contact and coincided to form a single magnitude, they did not make the whole any bigger”. (*GC I 2*, 316a30-33)

This paragraph is relevant to our discussion since it is here that Aristotle acknowledges the tension that results from his acceptance of two claims, central for his account of the infinite. First claim is the claim that a magnitude is infinitely divisible. Second claim is one that seems to have

been entirely uncontroversial in the Greek world, the claim that *what is continuous does not consist of points*.³ Aristotle offers the following solution.

For, since no point is contiguous to another point, magnitudes are divisible through and through in one sense, and yet not in another. When, however, it is admitted that a magnitude is divisible through and through, it is thought that there is a point not only anywhere, but also everywhere in it: hence it follows that the magnitude must be divided away into nothing. For there is a point everywhere within it, so that it consists either of contacts or of points. But it is only *in one sense* that the magnitude is divisible through and through, viz. in so far as there is one point *anywhere* within in and all its points are *everywhere* within it if you take them singly. (*GC I 2, 317a3-8*)

According to Aristotle, we must distinguish between *different senses* in which it could be said that a magnitude is divisible “through and through”. A magnitude *is not* divisible “through and through” in the sense that it could be divided *everywhere*⁴ (for then it would follow that it could be divided away into nothing). Note that it is not true that a line is *even potentially* divisible everywhere – Aristotle’s claim here is much stronger; it is the claim that this kind of dividing “through and through” is *impossible*.⁵ Still, a magnitude is infinitely divisible, because it can be divided *anywhere*.⁶ This is enough to establish infinite divisibility of a line, but strong enough to exclude the possibility of actually infinitely many points. We can divide a magnitude *at any point* (though not at *every point*), and what is left will again be divisible *at any point* and so on, *ad infinitum*.

³ Or, more generally, that no continuous entity is composed of entities of lower dimension, therefore, a line is not composed of points, a plane is not composed of lines etc. (*cf.* 231a20-b6)

⁴ Earlier in the chapter he states it more clearly: “It would seem to be impossible for a body to be divisible at all points simultaneously”. (*GC I 2, 216b21-23*)

⁵ And if it were potentially divisible in this way “then it [the division] might actually occur [...]. Consequently, nothing will remain and the body will have passed-away into what is incorporeal.” (*GC I 2. 316b23-26*)

⁶ *cf.* Lear, 1981, p. 200

Let us return to the potential-actual distinction. We wanted an answer to the question why Aristotle thought that a magnitude is not actually infinite by division, since that was the interpretation that made the disjunctive syllogism valid. Now, one interpretation seems to result from our analysis of the *GC* chapter. Since, a line cannot be *divided* “through and through”, the process of division will never have resulted in an actualization of *infinitely many* points or parts. This is the sense in which a magnitude is not actually but only potentially infinite by division – however long we divide it we will have actualized only finitely many points or parts. Still, as we have seen, any point could be actualized by dividing a magnitude at it (and any part can be actualized in this way), and in this sense, there are *potentially* infinitely many points or parts on a magnitude.

Having had a closer look at the premises, we can see how the conclusion follows. There is no actually infinite by addition (as shown in III 5)⁷, and there is only potentially infinite by division.

2. The infinite and a day

We have now arrived at a place where most interpreters look for clues for understanding Aristotle’s conception of the potentially infinite.

[I] But we must not construe potential existence in the way we do when we say that it is possible for this to be a statue – this will *be* a statue, but something infinite will not be in actuality. [II] Being is spoken of in many ways, and we say that the infinite is in the sense in which we say it is day or it is the games, because

⁷ In one sense there is also no potentially infinite by addition, but Aristotle does not argue for that until later (206b21-27) in the chapter.

one thing after another is always coming into existence. [III] For of these things too the distinction between potential and actual existence holds. [IV] We say that there are Olympic games, both in the sense that they may occur and that they are actually occurring. (Phys. III 6, 206a19-25)

In interpreting this passage, Hintikka (1966) argues that Aristotle accepts what he calls *the principle of plenitude*, according to which every genuine possibility is sometimes actualized. He uses this passage to prove that Aristotle's theory of the infinite does not provide a counterexample to the principle of plenitude, as is commonly thought. In short, he interprets the line "something infinite will not be in actuality" as saying, not that the infinite will never be *actual*, but that it never *is* (and therefore *is not actual*), in a way a *thing is*. Thus, according to Hintikka, the contrast between the infinite and a sculpture, and the parallel between the infinite and a day, is supposed to bring out that the infinite is not a *this (tode ti)*, as a statue eventually may be. It *exists, both potentially and actually*, in the same way in which a day exists, thus potentially when that it may occur, and actually when it is occurring. He concludes that "[i]n the *precise* sense, however, in which the infinite was found to exist potentially for Aristotle, it also exists actually". (Hintikka, 1966, p. 200)

Hintikka's conclusion seems to contradict my interpretation of the previous passage. According to my interpretation there *will never* be infinitely many actual points or parts on a line (*simply because* a line is not composed of points). A line is potentially infinitely divisible because there are points *anywhere* on a line, so that it could be divided at *any point*, but not *everywhere* on a line. This precisely is the sense in which the infinity of points and parts on a line is *only* potential. The potential existence of infinitely many points on a line is enough to ensure that the smallest magnitude does not exist, so that whatever magnitude we have we will be able to divide

it further. However, *a line does not consist of points*, and therefore it cannot be divided “through and through”, i.e. the infinity of points and parts can never be actualized.

However, there are independent reasons for believing that Hintikka’s interpretation does not do justice to Aristotle’s text. Although in some places (e.g. 206a23-25, 206b13-14) Aristotle seems to say that the infinite also exist actually, in other places he does say that it has only potential existence (e.g. the conclusion of the syllogism 206a16-18, and then again in 206b12-16). Hintikka has to explain these remarks as a “rather loose and inappropriate” (Hintikka, 1966, p. 199) way of speaking.⁸

It seems to me that most of the difficulties in interpreting this passage are explained away if we allow that Aristotle is talking not only about two senses of existence (actual and potential), but about the two senses in which the *infinite* may exist. Two respective senses of the infinite are:

- (a) the infinite in the sense of an infinite *process*
- (b) the infinite in the sense of a *product* of an infinite process (that is, either a thing with infinitely many parts or an infinitely extended thing)

With respect to both of these senses we could ask whether the infinite exist only potentially or also actually. It seems that if we take this distinction into account we could see why Aristotle can on the one side claim that the infinite exists *both potentially and actually*, and at the same time claim that the infinite is *only potential and never actual*. Let me elaborate.

⁸ Aristotle seems to deny that the infinite has actual existence also in *Metaphys.* VIII 6, 1048b14-17.

The infinite process can be either the process of addition or the process of division. To say that a process is infinite is to say that *it cannot be completed*, and both the process of addition and the process of division, if infinite, are infinite in this same sense. The distinction relevant for the discussion of processes is not that between the process of addition and division but between processes which form *convergent* series (either by addition or division) and those that form *non-convergent* series. Aristotle does recognize the importance of this distinction, and argues that:

In a way the infinite by addition is the same thing as the infinite by division. In a finite magnitude, the infinite by addition comes about in a way inverse to that of the other. For just as we see division going on *ad infinitum*, so we see the addition being made in the same proportion to what is already marked off. For if we take a determinate part of a finite magnitude and add another part determined by the same ratio (not taking in the same amount of the original whole), we shall not traverse the given magnitude. But, if we increase the ratio of the part, so as always to take in the same amount, we shall traverse the magnitude; for every finite magnitude is exhausted by means of any determinate quantity however small. (206b3-12)

Aristotle is saying here that we can use a given process of division, let us say one with the ratio $\frac{1}{2}$, to form an infinite process of addition, e.g. $\frac{1}{2} L + \frac{1}{4} L + \dots + \frac{1}{2^n} L$. This process of addition will never exhaust a given finite magnitude, so it could be continued without end. This is the way in which “the infinite by addition is the same thing as the infinite by division”. Clearly though, this process, even though it is a process of addition, does not show that there is an infinitely extended body. However, a non-convergent addition in which we would add equal units of length cannot be an infinite process, for even the greatest finite magnitude will be exhausted after finitely many additions. In this sense, the infinite process of addition does not exist, *even potentially*. (206b21). It is in this sense – of a process that cannot be completed – that the infinite

exists both potentially and actually; it exists potentially when it may take place, but it does not; actually when it is taking place.

When we think of the infinite in the sense (b), the distinction between the infinite by addition and the infinite by division is more relevant. The result of a process of infinite division of a magnitude would be *infinitely many actual points or parts*. On the other hand, the result of an infinite process of addition is *an infinitely extended magnitude*. The infinite in both of these senses is very different from the sense in which a process is infinite. Let us draw attention to just one of the disanalogies which will be of importance later. A process of addition or division is a series of *successive events* that do not exist *simultaneously*. The infinite by addition in the sense (b) would be *a body or a thing-like entity*, while the infinite by division in this sense would be *infinitely many thing-like entities* (parts or points) that all exist *simultaneously*.⁹

As far as division is concerned, as we have seen from *GC*, Aristotle thinks that the infinity of points on a magnitude (i.e. the infinite by division in the sense (b)) is a possibility that *can never be actualized*. Moreover, this fact explains why a process of division cannot be completed, simply, and *pace* Hintikka, because *there will always be possible divisions (and therefore points and parts) that will remain unactualized*.¹⁰ Thus, although Aristotle allows that an infinite process of division is actual when it is taking place, even as that actual process is taking place, the number of points and parts actualized remains finite. We can take the infinite by division in this sense to be only potential, because that is sufficient to account for the fact that a process of division cannot be completed (and therefore that there is *an infinite* process of division). As Lear

⁹ see Charlton, 1991, p.141

¹⁰ see Lear, 1982, p.191 for a similar account, also Bostock, 1973, p.39-40, who nonetheless takes the potential existence of the infinitude of points to be dependent on a process.

(1983, p.193) rightly notes, however a process of division ends, it will not end because all of the divisions had been made. Hence, the fact that a magnitude is *only potentially* infinite by division in the sense (b) ensures that it is *both actually and potentially* infinite by division in the sense (a).

Aristotle's discussion of the infinite by addition also seems to confirm this interpretation. A non-convergent process of addition does not exist even potentially (206b21) because universe has a finite, definite size. Therefore, because there is no infinite by addition in the sense (b), there is no infinite by addition in the sense (a) – every non-convergent process of addition will have to end after finitely many steps.¹¹

Returning to 206a19-25, we interpret it in the following way. In [I], we are taking Aristotle to be claiming that the infinite exist potentially, but the way in which it exists *potentially* is not the same as the way in which a statue exist *potentially*, the difference being that a statue may be actual, but the infinite may not. This seems to make better sense of this sentence than Hintikka's interpretation, because, if the only thing Aristotle wanted to bring out was that the infinite does not exist separately, it would have sufficed to say that it does not exist in a way a statue does (therefore saying that *the potential existence* of the infinite is not the same as *potential existence* of a statue would seem redundant). Also, it makes better sense of the previous paragraph, because there Aristotle is clearly saying that because (among other things) “a magnitude is not actually infinite [...] *the alternative then remains* that the infinite has a potential existence”.

¹¹ Note that Aristotle seems to think that it is necessary that a body be actually infinite by addition for a process of addition to be, even potentially, infinite. For in 206b21-23 he says the following. “But in respect of addition there cannot even potentially be an infinite which exceeds every assignable magnitude, *unless it is accidentally infinite in fulfillment*, as the physicist hold to be true of *the body* which is outside the world.” [my italics] This surely is not right, because we can think of universe as only potentially infinitely extended (for example, it is finite but it is always expanding), and then an infinite process of non-convergent addition could exist. This supports our interpretation of the disjunctive syllogism, because it explains why Aristotle is equating actually infinite with the infinite by addition.

(206a15-19). Now, we are taking [II] to be making a new point – being is spoken of in many ways, and in one of these ways the infinite exists in the sense in which a day or the games exist, as something that occurs, takes place. [III] and [IV] are then straightforward and do not contradict [I] – in this other sense, the infinite may be actual, just as a day or the games are.¹²

Aristotle goes on to say that:

The infinite exhibits itself in different ways – in time, in the generations of men, and in the division of magnitudes. For generally the infinite has this mode of existence: one thing is always being taken after another, and each thing that is taken is always finite, but always different. [...] But in spatial magnitudes, what is taken persists, while in the succession of time and of men it takes place by the passing away of these in such a way that the source of supply never gives out. (206a26-206b3)

It seems that Aristotle is making a distinction parallel to the one we have drawn. The infinite exhibits itself in different ways in time and in magnitude. The infinite with respect to time should presumably be understood as *a series of successive events* that are not all actual simultaneously, just as a whole day is never present. Thus, to say that time is infinite is to say that there is no end to this succession, that is, it is to say that some *process* cannot be completed (events pass away “in such a way that the source of supply never gives out”). In magnitude though “what is taken persists”. The way in which a magnitude can be infinite, Aristotle seems to be claiming, is different from the way in which time is infinite. It seems that this different sense that Aristotle is drawing our attention to is exactly our sense (b).

¹² Further down, Aristotle restates the point: “The infinite, then, exists in no other way, but in this way it does exist, potentially and by reduction. It exists in fulfillment in the sense in which we say ‘it is day’ or ‘it is the games’; and potentially as matter exists, not independently as what is finite does.” (206b13-16). This passage, perhaps, more obviously lends itself to the interpretation analogous to the one we have given for 206a19-25.

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