

Experimental Geometry - IDH3931-0009

Andrew Vince, Department of Mathematics

Of the images below, which are mathematically generated and which are photographs of natural objects? (Answers at the bottom of page 2. ¹)



Mathematics, and geometry in particular, provide powerful models for phenomena in the natural world. (The fractals among the images above are a recreational example.) The two broad themes of the course are particularly relevant mathematical models: **symmetry** and **chaos**. They are pervasive in nearly all the sciences (particle physics, crystallography, climatology, . . .). Although symmetry and chaos seem at opposite ends of a certain spectrum, systems that appear chaotic sometimes have hidden symmetry (the Penrose tiling in Figure 1, for example) and some systems that appear symmetric, upon closer scrutiny, exhibit extremely chaotic behavior (the Lorenz attractor in Figure 1, for example). Five particular topics in geometry have been chosen to illustrate the two themes.

The goal of the course is for students to finish with an understanding of the following five mathematical subjects, each shedding light on the themes mentioned above. That understanding may not be completely rigorous - a graduate course could be offered on any one of these subjects - but the students should come away with a concrete appreciation of the mathematical significance of each.

1. **plane geometries** (Fig. a)
2. **polytopes** (Fig. b)
3. **tessellations** (Fig. c)
4. **fractals** (Fig. d)
5. **dynamical systems** (Fig. e)

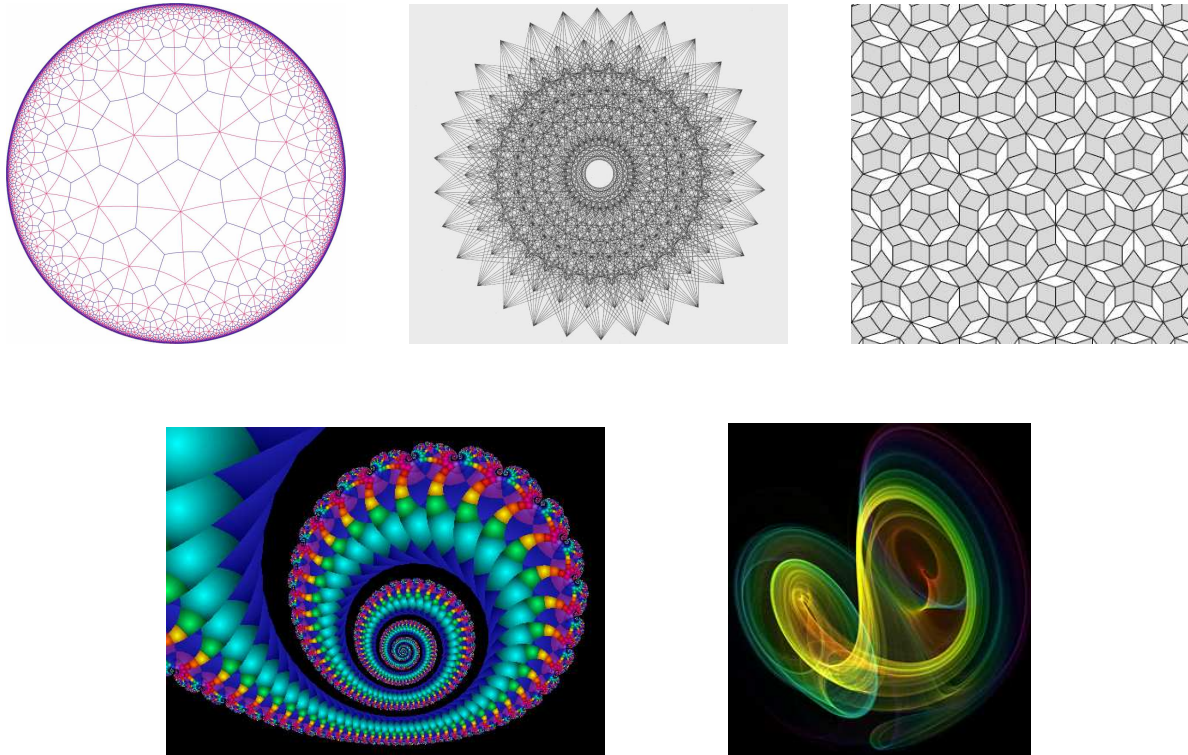


Figure 1: (a) hyperbolic plane, (b) 4-D polytope, (c) Penrose tiling, (d) self-similar fractal, (e) Lorenz attractor

The method of instruction is geometric experimentation. The students will use the software: GEOMETER'S SKETCHPAD, MAPLE, and/or MATHEMATICA - all readily available on our computer network - to make discoveries that add to their knowledge of the topics. This approach is not new. The universities of Minnesota and Maryland, for example, have centers that emphasize visualization in geometry. It is not, however, the method used in mathematics courses at UF. In addition to the instructor's input to the course, each student will present his/her own discoveries in class.

Students will be evaluated on the basis of reports on their experiments and on their classroom presentations.

Prerequisites are a secondary school course in geometry, a familiarity with computers (but not necessarily a knowledge of the particular software to be used in the course), and a willingness to trust ones own intellect in drawing conclusions from the geometric experiments.

¹The second image is a coral, the fourth a broccoli; the other two are mathematically generated.