Numerical methods for locating small dielectric inhomogeneities based on asymptotics of perturbed fields and application of selected incoming waves

Dielectric inhomogeneities of small diameter cause a perturbation of the ambient electromagnetic fields. It is easily conceivable that measuring those perturbations could provide information relevant to the identification and characterization of the inhomogeneities. Engineers have been using that idea for a long time, for example for the non destructive testing of materials. Another field of interest for applications is medical imaging via magnetic resonance.

A mathematical study of the perturbation of electromagnetic fields due to inhomogeneities was developed among others, by H. Ammari, S. Moskow, D. Volkov, M. Vogelius.

In the case of a medium whose boundary can be subjected to a time sinusoidal electric field, asymptotic expansions of the perturbed magnetic field were obtained. The small parameter was the relative diameter of the inhomogeneities. It was claimed that those asymptotic formulas could be used for solving the inverse problem consisting of reconstructing the imperfections based on measurements of perturbed fields. Some numerical simulations were conducted in an effort to demonstrate the efficiency of the reconstruction algorithm. However, those simulations were based on synthetic data that did not take into account higher errors in the asymptotic approximation induced by working with high frequencies or highly oscillatory currents in amplitude versus space variables. We propose to discus s these hurdles and we show on a numerical simulation how they can be overcome.

Asymptotic formulas were also derived for the far field pattern of the scattered wave produced by an incoming plane wave impinging a set of inhomogeneities. The possible applications that we bear in mind in this context range from radar imaging to the detection of mines. We propose a reconstruction algorithm based on projections on three different planes. In our algorithm, the incoming waves all have the same frequency. We assume that angle of observation and direction of propagation of the incoming plane wave can be chosen at will. In terms of computational time this method amounts to solving three times a two-dimensional problem. This is a great saving as compared to a full fledged three-dimensional approach making use of spherical harmonics.