March 4th: Computational Methods / Optimization

Dr. Stephen Wright, *University of Wisconsin*

Optimization Algorithms for Compressed Sensing.

Compressed sensing, an area of tremendous recent research activity in applied mathematics, statistics, and engineering, is the problem of finding a sparse approximate solution (that is, one with few nonzero elements) to an underdetermined system of linear equations. Under certain assumptions, the problem can be approximated well by a convex optimization formulation. Optimization methods for linear and quadratic programming, suitably adapted to the structure of the application, can then be used to solve it. Such methods are among the most successful approaches yet found for compressed sensing applications. In this talk, we discuss the problem, outline some of the successful algorithms, and present some computational results.

Minah Oh, *University of Florida*

Multigrid in a weighted space arising from axisymmetric electromagnetic.

Consider the space of two-dimensional vector functions whose components and curl are square integrable with respect to the degenerate weight given by the radial variable. This space arises naturally when modeling electromagnetic problems under axial symmetry and performing a dimension reduction via cylindrical coordinates. I will present a result on the convergence of the multigrid V-cycle (at a rate uniform with respect to the meshsize) when applied to the inner product in this space. I will also present several interesting ingredients we need for our convergence analysis, such as the approximation properties of certain commuting projectors in weighted norms, and a superconvergence result for a dual mixed method in weighted spaces. I will also present numerical results providing empirical support to the theoretical results.

Ali Al-Sharadqah, *University of Alabama, Birmingham*

Error analysis for circle fitting algorithms.

Fitting circles and circular arcs to observed points is one of the basic tasks in pattern recognition and computer vision, nuclear physics, and other areas. Many algorithms have been developed that fit circles to data. Some minimize the geometric distances from the circle to the data points (geometric fit). Others minimize various approximate `algebraic' distances, they are called algebraic fits.

Geometric fit is commonly regarded as the most accurate, but it can only be implemented by iterative schemes that are computationally intensive and subject to occasional divergence. Algebraic fits are faster but presumably less precise. At the same time the assessments on their accuracy are solely based on practical experience, no one has performed a detailed theoretical comparison of the accuracy of various circle fits.

In our paper, we present a detailed error analysis for all popular circle fitting methods (geometric fit, Kasa fit, Pratt fit, and Taubin fit). We apply this approach to the circle fits to show exactly why and by how much the geometric fit outperforms the algebraic fits in accuracy; we also compare the precision of different algebraic fits. Our analysis allows us to construct a new algebraic (non-iterative) circle fitting algorithm that outperforms all the existing methods, including the (previously regarded as unsurpassable) geometric fit.

Sergey Yakovlev, *Rensselaer Polytechnic Institute*

A Central DG Method for Hamilton-Jacobi Equations.

Hamilton-Jacobi equations provide important mathematical models for many areas such as optimal control, computer vision, and geometric optics. In this talk I will discuss a new central discontinuous Galerkin method for solving such equations. Theoretical results on accuracy and stability have been established when the Hamiltonian is linear. And the performance of the proposed method will be demonstrated through a sequence of numerical examples with linear/nonlinear, convex/non-convex Hamiltonians.

William Hager, *University of Florida*

An Affine-scaling Interior-point CBB Method and Applications.

An affine-scaling algorithm for optimization problems with a single linear equality constraint and box restrictions is developed. The algorithm has the property that each iterate lies in the relative interior of the feasible set. The search direction is obtained by approximating the Hessian of the objective function in Newton's method by a multiple of the identity matrix The approximation given by the cyclic Barzilai-Borwein (CBB) formula yields the ASL-CBB scheme (affine scaling interior point method for linear constraints). Global convergence is established for a nonmonotone line search. The algorithm is particularly well suited for optimization problems where the Hessian of the objective function is a large, dense, and possibly ill-conditioned matrix. Application to Support Vector Machines (SVM) and to Positron Emission Tomography (PET) are also considered.

Marta D'Elia, *Emory University*

Numerical Methods for including Measured Velocities in an incompressible Stokes Simulation.

Efficient and reliable imaging techniques allow to collect a huge amount of biomedical data. Given some measurements of the velocity blood field inside a vessel, in correspondence of some internal surfaces, we want to recover the blood flow in the whole vessel. The goal of this work is to include these measurements, affected by noise, in the numerical simulations combining the data and the dynamical principles governing the flow described by the incompressible Stokes equations; this process is also known as Data Assimilation. In this work we consider several techniques: methods arising from control theory for Computational Fluid Dynamics (where the difference between the predicted data and the data itself is minimized subject to the state equations, Optimize-Discretize and Discretize-Optimize approaches are performed), methods for inverse problems (where some techniques for the regularization of this potentially ill-posed problem are used), Dynamic Relaxation (a method used for Data Assimilation in meteorology) and splitting-domain techniques. Numerical tests are performed in the steady and unsteady case with promising results in view of the generalization of these approaches to the nonlinear Navier-Stokes equations.

Shuguang Tan, *University of Florida*

Multigrid for Hybridized Mixed Method.

We consider the application of a variable V-cycle multigrid algorithm for the hybridized mixed method for second order elliptic boundary value problems. Our algorithm differs from previous works on multigrid for the mixed method in that it is targeted at efficiently solving the matrix system for the Lagrange multiplier of the method. Since the mixed method is best implemented by first solving for the Lagrange multiplier and recovering the remaining unknowns locally, our algorithm is more useful in practice. The critical ingredient in the algorithm is a suitable intergrid transfer operator. We design such an operator and prove mesh independent convergence of the variable Vcycle algorithm. Numerical experiments indicating the asymptotically optimal performance of our algorithm, as well as the failure of certain seemingly plausible intergrid transfer operators, are presented.

Melih Ozlem, *Rensselaer Polytechnic Institute*

Numerical Modeling of Void Collapse

Motivated by the need for an improved understanding of a prominent mechanism of the generation of hot spots (discrete sites of preferential reaction), which play a crucial role in the ignition of a shocked heterogeneous explosive, a model of shock-induced void collapse in a solid material is examined numerically. Specifically, an axisymmetric configuration consisting of a single gas bubble in a solid matrix is considered. The mathematical model is a system of hyperbolic PDEs, the Euler equations, supplemented by nonideal equations of state for the solid and gas constituents. A mixture formulation is introduced, and the interface is treated as a zone of finite thickness extending over a few computational cells. A finite-volume numerical strategy is employed; it incorporates adaptive mesh refinement and is based on a variant of the Godunov scheme modified to suppress nonphysical instabilities in the vicinity of shocks and interfaces. Complete histories of bubble collapse induced by an incident shock are presented for a variety of bubble shapes. Hydrodynamic features produced by the shock-bubble interaction are carefully followed, and special attention is paid to mechanisms leading to the evolution of regions of high temperature.

Dung Phan, *University of Florida*

Solving graph bisection minimization problems using convex quadratic relaxations.

In this talk, we present an exact algorithm for solving the graph partitioning problem using the branch and bound techniques. A continuous quadratic formulation is investigated. Lower bounds for the branch and bound algorithm are obtained by replacing the concave part of the objective function by a linear underestimate. Preliminary numerical results are reported.

POSTER PRESENTATIONS

Gloriell M. Cardona-Meléndez, *University of Puerto Rico at Cayey*

Statistical analysis of the relationship between size and clutch size for Mona Island iguana (Cyclura cornuta stejnegeri).

The Mona Island iguana*, Cyclura cornuta stejnegeri,* is an endangered species since 1973. Its population is decreasing by the predation of introduced mammals, such as cats and pigs. In addition, the incorporation of *Casuarina equisetifolia (Australian pine)* has affected the nesting site. Currently, a research effort for developing a Population Viability Analysis for Mona Island Iguana has been undertaken in order to predict the future size of the population of this species and to evaluate current conservation efforts. As part of this effort, data relating the size of a female (snout-vent length –SVL-, tail length and body mass) and the number of eggs laid in each clutch are analyzed using Generalized Linear Models based on the Poisson distribution. All calculations were done with the statistical software R. The method used consists in analyzing the data using the software package R. From the fitted models, it can be seen that bigger sizes are related to bigger clutch sizes.

Amy Mihnea, *Florida Atlantic University*

Clustering and Image Compression.

I used partitioning around medoids for the rows of an image to reduce its entropy. Then I used the reduced image and did the same thing on the columns. If we do these in the opposite order the results are not significantly different and we could use this method to get cheaper data from which we can recover the original one.